

# Algebraic Number Theory

(PARI-GP version 2.9.0)

## Binary Quadratic Forms

create  $ax^2 + bxy + cy^2$  (distance  $d$ )      `Qfb(a,b,c,{d})`  
reduce  $x$  ( $s = \sqrt{D}$ ,  $l = \lfloor s \rfloor$ )      `qfbred(x,{flag},{D},{l},{s})`  
return  $[y, g]$ ,  $g \in \text{SL}_2(\mathbf{Z})$ ,  $y = g \cdot x$  reduced      `qfbreds12(x)`  
composition of forms       $x*y$  or `qfbnucomp(x,y,l)`  
 $n$ -th power of form       $x^n$  or `qfbnupow(x,n)`  
composition without reduction      `qfbcomprow(x,y)`  
 $n$ -th power without reduction      `qfbpowrow(x,n)`  
prime form of disc.  $x$  above prime  $p$       `qfbprimeform(x,p)`  
class number of disc.  $x$       `qfbclassno(x)`  
Hurwitz class number of disc.  $x$       `qfbhclassno(x)`  
Solve  $Q(x,y) = p$  in integers,  $p$  prime      `qfbsolve(Q,p)`

## Quadratic Fields

quadratic number  $\omega = \sqrt{x}$  or  $(1 + \sqrt{x})/2$       `quadgen(x)`  
minimal polynomial of  $\omega$       `quadpoly(x)`  
discriminant of  $\mathbf{Q}(\sqrt{D})$       `quaddisc(x)`  
regulator of real quadratic field      `quadregulator(x)`  
fundamental unit in real  $\mathbf{Q}(x)$       `quadunit(x)`  
class group of  $\mathbf{Q}(\sqrt{D})$       `quadclassunit(D,{flag},{t})`  
Hilbert class field of  $\mathbf{Q}(\sqrt{D})$       `quadhilbert(D,{flag})`  
... using specific class invariant ( $D < 0$ )      `polclass(D,{inv})`  
ray class field modulo  $f$  of  $\mathbf{Q}(\sqrt{D})$       `quadray(D,f,{flag})`

## General Number Fields: Initializations

The number field  $K = \mathbf{Q}[X]/(f)$  is given by irreducible  $f \in \mathbf{Q}[X]$ .  
A  $nf$  computes a maximal order and allows operations on elements and ideals. A  $bnf$  adds class group and units. A  $bnr$  is attached to ray class groups and class field theory. A  $rnf$  is attached to relative extensions  $L/K$ .

init number field structure  $nf$       `nfinit(f,{flag})`  
known integer basis  $B$       `nfinit([f,B])`  
order maximal at  $vp = [p_1, \dots, p_k]$       `nfinit([f,vp])`  
order maximal at all  $p \leq P$       `nfinit([f,P])`  
certify maximal order      `nfcertify(nf)`

### nf members:

a monic  $F \in \mathbf{Z}[X]$  defining  $K$        $nf.pol$   
number of real/complex places       $nf.r1/r2/sign$   
discriminant of  $nf$        $nf.disc$   
 $T_2$  matrix       $nf.t2$   
complex roots of  $F$        $nf.roots$   
integral basis of  $\mathbf{Z}_K$  as powers of  $\theta$        $nf.zk$   
different/codifferent       $nf.diff, nf.codiff$   
index  $[\mathbf{Z}_K : \mathbf{Z}[X]/(F)]$        $nf.index$   
recompute  $nf$  using current precision       $nf.newprec(nf)$   
init relative  $rnf$   $L = K[Y]/(g)$        $rnfinit(nf,g)$   
init  $bnf$  structure       $bnfinit(f,{flag})$

### bnf members:

underlying  $nf$        $bnf.nf$   
classgroup       $bnf.clgp$   
regulator       $bnf.reg$   
fundamental/torsion units       $bnf.fu, bnf.tu$   
compress a  $bnf$  for storage       $bnf.compress(bnf)$   
recover a  $bnf$  from compressed  $bnfz$        $bnfinit(bnfz)$   
add  $S$ -class group and units, yield  $bnfS$        $bnfsunit(bnf,S)$   
init class field structure  $bnr$        $bnrinit(bnf,m,{flag})$

### bnr members:

same as  $bnf$ , plus  
underlying  $bnf$        $bnr.bnf$   
big ideal structure       $bnr.bid$   
modulus       $bnr.mod$   
structure of  $(\mathbf{Z}_K/m)^*$        $bnr.zkst$

## Basic Number Field Arithmetic (nf)

Elements are `t_INT`, `t_FRAC`, `t_POL`, `t_POLMOD`, or `t_COL` (on integral basis  $nf.zk$ ). Basic operations (prefix `nfelt`): (`nfelt`)`add`, `mul`, `pow`, `div`, `diveuc`, `mod`, `divrem`, `val`, `trace`, `norm`  
express  $x$  on integer basis      `nfalgtobasis(nf,x)`  
express element  $x$  as a polmod      `nfbasistoalg(nf,x)`  
complex embeddings of `t_POLMOD`  $x$       `conjvec(x)`  
reverse polmod  $a = A(X) \bmod T(X)$       `modreverse(a)`  
integral basis of field def. by  $f = 0$       `nfbasis(f)`  
field discriminant of field  $f = 0$       `nfdisc(f)`  
smallest poly defining  $f = 0$  (slow)      `polredabs(f,{flag})`  
small poly defining  $f = 0$  (fast)      `polredbest(f,{flag})`  
random Tschirnhausen transform of  $f$       `poltschirnhaus(f)`  
 $\mathbf{Q}[x]/(f) \subset \mathbf{Q}[x]/(g)$  ? Isomorphic?      `nfisincl(f,g), nfisisom`  
compositum of  $\mathbf{Q}[X]/(f)$ ,  $\mathbf{Q}[X]/(g)$       `polcompositum(f,g,{flag})`  
compositum of  $K[X]/(f)$ ,  $K[X]/(g)$       `nfcompositum(nf,f,g,{flag})`  
splitting field of  $K$  (degree divides  $d$ )      `nfsplitting(nf,{d})`  
subfields (of degree  $d$ ) of  $nf$       `nfsubfields(nf,{d})`  
 $d$ -th degree subfield of  $\mathbf{Q}(\zeta_n)$       `polsubcyclo(n,d,{v})`  
roots of unity in  $nf$       `nfrootsof1(nf)`  
roots of  $g$  belonging to  $nf$       `nfroots({nf},g)`  
factor  $g$  in  $nf$       `nfactor(nf,g)`  
factor  $g \bmod$  prime  $pr$  in  $nf$       `nfactormod(nf,g,pr)`  
conjugates of a root  $\theta$  of  $nf$       `nfgaloisconj(nf,{flag})`  
apply Galois automorphism  $s$  to  $x$       `nfgaloisapply(nf,s,x)`  
quadratic Hilbert symbol (at  $p$ )      `nfhilbert(nf,a,b,{p})`

### Linear and algebraic relations

poly of degree  $\leq k$  with root  $x \in \mathbf{C}$       `algdep(x,k)`  
alg. dep. with pol. coeffs for series  $s$       `seralgdep(s,x,y)`  
small linear rel. on coords of vector  $x$       `lindexp(x)`

### Dedekind Zeta Function $\zeta_K$ , Hecke $L$ series

$R = [c, w, h]$  in initialization means we restrict  $s \in \mathbf{C}$  to domain  $|\Re(s) - c| < w$ ,  $|\Im(s)| < h$ ;  $R = [w, h]$  encodes  $[1/2, w, h]$  and  $[h]$  encodes  $R = [1/2, 0, h]$  (critical line up to height  $h$ ).

$\zeta_K$  as Dirichlet series,  $N(I) < b$       `dirzetak(nf,b)`  
init  $\zeta_K^{(k)}(s)$  for  $k \leq n$       `L = lfuninit(bnf,R,{n=0})`  
compute  $\zeta_K(s)$  ( $n$ -th derivative)      `lfun(L,s,{n=0})`  
compute  $\Lambda_K(s)$  ( $n$ -th derivative)      `lfunlambda(L,s,{n=0})`

init  $L_K^{(k)}(s, \chi)$  for  $k \leq n$       `L = lfuninit([bnr,chi],R,{n=0})`  
compute  $L_K(s, \chi)$  ( $n$ -th derivative)      `lfun(L,s,{n})`  
Artin root number of  $K$       `bnrrootnumber(bnr,chi,{flag})`  
 $L(1, \chi)$ , for all  $\chi$  trivial on  $H$       `bnrL1(bnr,{H},{flag})`

## Class Groups & Units (bnf, bnr)

Class field theory data  $a_1, \{a_2\}$  is usually  $bnr$  (ray class field),  $bnr, H$  (congruence subgroup) or  $bnr, \chi$  (character on `bnr.clgp`). Any of these define a unique abelian extension of  $K$ .  
remove GRH assumption from  $bnf$       `bnfcertify(bnf)`  
expo. of ideal  $x$  on class gp      `bnfisprincipal(bnf,x,{flag})`  
expo. of ideal  $x$  on ray class gp      `bnrisprincipal(bnr,x,{flag})`  
expo. of  $x$  on fund. units      `bnfisunit(bnf,x)`  
as above for  $S$ -units      `bnfissunit(bnfs,x)`

signs of real embeddings of  $bnf.fu$       `bnfsignunit(bnf)`  
narrow class group      `bnfnarrow(bnf)`  
**Class Field Theory**  
ray class number for modulus  $m$       `bnrclassno(bnf,m)`  
discriminant of class field      `bnrdisc(a_1,{a_2})`  
ray class numbers,  $l$  list of moduli      `bnrclassolist(bnf,l)`  
discriminants of class fields      `bnrdisclist(bnf,l,{arch},{flag})`  
decode output from `bnrdisclist`      `bnfdecodemodule(nf,fa)`  
is modulus the conductor?      `bnrisconductor(a_1,{a_2})`  
is class field ( $bnr, H$ ) Galois over  $K^G$       `bnrisgalois(bnr,G,H)`  
action of automorphism on `bnr.gen`      `bnrgaloismatrix(bnr,aut)`  
apply `bnrgaloismatrix`  $M$  to  $H$       `bnrgaloisapply(bnr,M,H)`  
characters on `bnr.clgp` s.t.  $\chi(g_i) = e(v_i)$       `bnrchar(bnr,g,{v})`  
conductor of character  $\chi$       `bnrconductor(bnr,chi)`  
conductor of extension      `bnrconductor(a_1,{a_2},{flag})`  
conductor of extension  $K[Y]/(g)$       `rnfconductor(bnf,g)`  
Artin group of extension  $K[Y]/(g)$       `rnfnormgroup(bnr,g)`  
subgroups of  $bnr$ , index  $\leq b$       `subgrouplist(bnr,b,{flag})`  
rel. eq. for class field def'd by  $sub$       `rnfkummer(bnr,sub,{d})`  
same, using Stark units (real field)      `bnrstark(bnr,sub,{flag})`  
is a an  $n$ -th power in  $K_v$  ?      `nfislocalpower(nf,v,a,n)`  
cyclic  $L/K$  satisf. local conditions      `nfgrunwaldwang(nf,P,D,pl)`

### Logarithmic class group

logarithmic  $\ell$ -class group      `bnflog(bnf,l)`  
 $[\tilde{e}(F_v/Q_p), \tilde{f}(F_v/Q_p)]$       `bnflogef(bnf,pr)`  
 $\exp \deg_F(A)$       `bnflogdegree(bnf,A,l)`  
is  $\ell$ -extension  $L/K$  locally cyclotomic      `rnfislocalcyclo(rmf)`

### Ideals:

elements, primes, or matrix of generators in HNF  
is  $id$  an ideal in  $nf$  ?      `nfisideal(nf,id)`  
is  $x$  principal in  $bnf$  ?      `bnfisprincipal(bnf,x)`  
give  $[a, b]$ , s.t.  $a\mathbf{Z}_K + b\mathbf{Z}_K = x$       `idealtwoelt(nf,x,{a})`  
put ideal  $a$  ( $a\mathbf{Z}_K + b\mathbf{Z}_K$ ) in HNF form      `idealhnf(nf,a,{b})`  
norm of ideal  $x$       `idealnrm(nf,x)`  
minimum of ideal  $x$  (direction  $v$ )      `idealmin(nf,x,v)`  
LLL-reduce the ideal  $x$  (direction  $v$ )      `idealred(nf,x,{v})`

### Ideal Operations

add ideals  $x$  and  $y$       `idealadd(nf,x,y)`  
multiply ideals  $x$  and  $y$       `idealmul(nf,x,y,{flag})`  
intersection of ideals  $x$  and  $y$       `idealintersect(nf,x,y,{flag})`  
 $n$ -th power of ideal  $x$       `idealpow(nf,x,n,{flag})`  
inverse of ideal  $x$       `idealinu(nf,x)`  
divide ideal  $x$  by  $y$       `idealdiv(nf,x,y,{flag})`  
Find  $(a, b) \in x \times y$ ,  $a + b = 1$       `idealaddtoone(nf,x,{y})`  
coprime integral  $A, B$  such that  $x = A/B$       `idealnumden(nf,x)`

### Primes and Multiplicative Structure

factor ideal  $x$  in  $\mathbf{Z}_K$       `idealfactor(nf,x)`  
expand ideal factorization in  $K$       `idealfactorback(nf,f,{e})`  
expand elt factorisation in  $K$       `nffactorback(nf,f,{e})`  
decomposition of prime  $p$  in  $\mathbf{Z}_K$       `idealprimedec(nf,p)`  
valuation of  $x$  at prime ideal  $pr$       `idealval(nf,x,pr)`  
weak approximation theorem in  $nf$       `idealchinese(nf,x,y)`  
 $a \in K$ , s.t.  $v_p(a) = v_p(x)$  if  $v_p(x) \neq 0$       `idealappr(nf,x)`  
 $a \in K$  such that  $(a \cdot x, y) = 1$       `idealcoprime(nf,x,y)`  
give  $bid$  = structure of  $(\mathbf{Z}_K/id)^*$       `idealstar(nf,id,{flag})`  
structure of  $(1 + \mathfrak{p})/(1 + \mathfrak{p}^k)$       `idealprincipalunits(nf,pr,k)`  
discrete log of  $x$  in  $(\mathbf{Z}_K/bid)^*$       `ideallog(nf,x,bid)`

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**idealstar** of all ideals of norm  $\leq b$       **ideallist**( $nf, b, \{flag\}$ )  
 add Archimedean places      **ideallistarch**( $nf, b, \{ar\}, \{flag\}$ )  
 init **modpr** structure      **nfmodprinit**( $nf, pr$ )  
 project  $t$  to  $\mathbf{Z}_K/pr$       **nfmodpr**( $nf, t, modpr$ )  
 lift from  $\mathbf{Z}_K/pr$       **nfmodprlift**( $nf, t, modpr$ )

## Galois theory over $\mathbf{Q}$

Galois group of field  $\mathbf{Q}[x]/(f)$       **polgalois**( $f$ )  
 initializes a Galois group structure  $G$       **galoisinit**( $pol, \{den\}$ )  
 action of  $p$  in **nfgaloisconj** form      **galoispermopol**( $G, \{p\}$ )  
 identify as abstract group      **galoisidentify**( $G$ )  
 export a group for GAP/MAGMA      **galoisexport**( $G, \{flag\}$ )  
 subgroups of the Galois group  $G$       **galoissubgroups**( $G$ )  
 is subgroup  $H$  normal?      **galoisisnormal**( $G, H$ )  
 subfields from subgroups      **galoissubfields**( $G, \{flag\}, \{v\}$ )  
 fixed field      **galoisfixedfield**( $G, perm, \{flag\}, \{v\}$ )  
 Frobenius at maximal ideal  $P$       **idealfrobenius**( $nf, G, P$ )  
 ramification groups at  $P$       **idealramgroups**( $nf, G, P$ )  
 is  $G$  abelian?      **galoisisabelian**( $G, \{flag\}$ )  
 abelian number fields/ $\mathbf{Q}$       **galoissubcyclo**( $\mathbf{N}, H, \{flag\}, \{v\}$ )  
 query the **galpol** package      **galoisgetpol**( $a, b, \{s\}$ )

## Relative Number Fields (rnf)

Extension  $L/K$  is defined by  $T \in K[x]$ .  
 absolute equation of  $L$       **rnfequation**( $nf, T, \{flag\}$ )  
 is  $L/K$  abelian?      **rnfisabelian**( $nf, T$ )  
 relative **nfalttobasis**      **rnfalttobasis**( $rnf, x$ )  
 relative **nfbasistoalg**      **rnfbasistoalg**( $rnf, x$ )  
 relative **idealhnf**      **rnfidealhnf**( $rnf, x$ )  
 relative **idealmul**      **rnfidealmul**( $rnf, x, y$ )  
 relative **idealtwoelt**      **rnfidealtwoelt**( $rnf, x$ )

## Lifts and Push-downs

absolute  $\rightarrow$  relative repres. for  $x$       **rnfeltabstorel**( $rnf, x$ )  
 relative  $\rightarrow$  absolute repres. for  $x$       **rnfeltreltoabs**( $rnf, x$ )  
 lift  $x$  to the relative field      **rnfeltup**( $rnf, x$ )  
 push  $x$  down to the base field      **rnfeltdown**( $rnf, x$ )  
 idem for  $x$  ideal: (**rnfideal**)**reltoabs**, **abstorel**, **up**, **down**

## Norms and Trace

relative norm of element  $x \in L$       **rnfeltnorm**( $rnf, x$ )  
 relative trace of element  $x \in L$       **rnfelttrace**( $rnf, x$ )  
 absolute norm of ideal  $x$       **rnfidealnrmabs**( $rnf, x$ )  
 relative norm of ideal  $x$       **rnfidealnrmrel**( $rnf, x$ )  
 solutions of  $N_{K/\mathbf{Q}}(y) = x \in \mathbf{Z}$       **bnfisintnorm**( $bnf, x$ )  
 is  $x \in \mathbf{Q}$  a norm from  $K$ ?      **bnfisnorm**( $bnf, x, \{flag\}$ )  
 initialize  $T$  for norm eq. solver      **rnfisnorminit**( $K, pol, \{flag\}$ )  
 is  $a \in K$  a norm from  $L$ ?      **rnfisnorm**( $T, a, \{flag\}$ )  
 initialize  $t$  for Thue equation solver      **thueinit**( $f$ )  
 solve Thue equation  $f(x, y) = a$       **thue**( $t, a, \{sol\}$ )  
 characteristic poly. of  $a$  mod  $T$       **rnfcharpoly**( $nf, T, a, \{v\}$ )

## Factorization

factor ideal  $x$  in  $L$       **rnfidealfactor**( $rnf, x$ )  
 $[S, T]: T_{i,j} \mid S_i; S$  primes of  $K$  above  $p$       **rnfidealprimedec**( $rnf, p$ )

## Maximal order $\mathbf{Z}_L$ as a $\mathbf{Z}_K$ -module

relative **polredbest**      **rnfpolredbest**( $nf, T$ )  
 relative Dedekind criterion, prime  $pr$       **rnfdedekind**( $nf, T, pr$ )  
 discriminant of relative extension      **rnfdisc**( $nf, T$ )  
 pseudo-basis of  $\mathbf{Z}_L$       **rnfpsedobasis**( $nf, T$ )  
**General  $\mathbf{Z}_K$ -modules:**  $M = [\text{matrix, vec. of ideals}] \subset L$   
 relative HNF / SNF      **nfhnf**( $nf, M$ ), **nfsnf**  
 multiple of det  $M$       **nfdetint**( $nf, M$ )  
 HNF of  $M$  where  $d = \text{nfdetint}(M)$       **nfhnfmod**( $x, d$ )  
 reduced basis for  $M$       **rnflllgram**( $nf, T, M$ )  
 determinant of pseudo-matrix  $M$       **rnfdet**( $nf, M$ )  
 Steinitz class of  $M$       **rnfsteinitz**( $nf, M$ )  
 $\mathbf{Z}_K$ -basis of  $M$  if  $\mathbf{Z}_K$ -free, or 0      **rnfhnfbasis**( $bnf, M$ )  
 $n$ -basis of  $M$ , or  $(n+1)$ -generating set      **rnfbasis**( $bnf, M$ )  
 is  $M$  a free  $\mathbf{Z}_K$ -module?      **rnfisfree**( $bnf, M$ )

## Associative Algebras

$A$  is a general associative algebra given by a mult. table  $mt$  (over  $\mathbf{Q}$  or  $\mathbf{F}_p$ ); represented by  $al$  from **algtbleinit**.  
 create  $al$  from  $mt$  (over  $\mathbf{F}_p$ )      **algtbleinit**( $mt, \{p=0\}$ )  
 group algebra  $\mathbf{Q}[G]$  (or  $\mathbf{F}_p[G]$ )      **alggroup**( $G, \{p=0\}$ )

## Properties

is  $(mt, p)$  OK for **algtbleinit**?      **algisassociative**( $mt, \{p=0\}$ )  
 multiplication table  $mt$       **algmtable**( $al$ )  
 multiplication table over center      **algrelmtable**( $al$ )  
 dimension of  $A$  over prime subfield      **algabsdim**( $al$ )  
 characteristic of  $A$       **algchar**( $al$ )  
 is  $A$  commutative?      **algiscommutative**( $al$ )  
 is  $A$  simple?      **algissimple**( $al$ )  
 is  $A$  semi-simple?      **algissemisimple**( $al$ )  
 is  $A$  ramified? (at place  $v$ )      **algisramified**( $al, \{v\}$ )  
 is  $A$  split? (at place  $v$ )      **algissplit**( $al, \{v\}$ )  
 center of  $A$       **algcenter**( $al$ )  
 Jacobson radical of  $A$       **algradical**( $al$ )  
 radical  $J$  and simple factors of  $A/J$       **algdecomposition**( $al$ )  
 simple factors of semi-simple  $A$       **algsimpledec**( $al$ )

## Operations on algebras

create  $A/I$ ,  $I$  two-sided ideal      **algquotient**( $al, I, \{flag=0\}$ )  
 create  $A_1 \otimes A_2$       **algtensor**( $al1, al2$ )  
 create subalgebra from basis  $B$       **algsubalg**( $al, B$ )  
 $\dots$  from orthogonal central idempotents  $e$       **algcentralproj**( $al, e$ )  
 prime subalgebra of semi-simple  $A$  over  $\mathbf{F}_p$       **algprimesubalg**( $al$ )  
 lattice generated by cols. of  $M$       **alglathnf**( $al, M$ )

## Operations on elements

$a+b, a-b, -a$       **algadd**( $al, a, b$ ), **algsub**, **algneg**  
 $a \times b, a \times a$       **algmul**( $al, a, a$ ), **algsqr**  
 $a^n, a^{-1}$       **algpow**( $al, a, n$ ), **alginv**  
 is  $x$  invertible? (then set  $z = x^{-1}$ )      **algisinv**( $al, x, \{\&z\}$ )  
 find  $z$  such that  $x \times z = y$       **algdivl**( $al, x, y$ )  
 find  $z$  such that  $z \times x = y$       **algdivr**( $al, x, y$ )  
 does  $z$  s.t.  $x \times z = y$  exist? (set it)      **algisdivl**( $al, x, y, \{\&z\}$ )  
 matrix of  $v \mapsto x \cdot v$       **algleftmtable**( $al, x$ )  
 absolute norm      **algnorm**( $al, x$ )  
 absolute trace      **algtrace**( $al, x$ )  
 absolute char. polynomial      **algcharpoly**( $al, x$ )  
 given  $a \in A$  and polynomial  $T$ , return  $T(a)$       **algpoleval**( $al, T, a$ )  
 random element in a box      **algrandom**( $al, b$ )

## Central Simple Algebras

$A$  is a central simple algebra over a number field  $K$ ; represented by  $al$  from **alginitt**;  $K$  is given by a  $nf$  structure.  
 create CSA from data      **alginitt**( $B, C, \{v\}, \{flag=0\}$ )  
 multiplication table over  $K$        $B = K, C = mt$   
 cyclic algebra  $(L/K, \sigma, b)$        $B = rnf, C = [\text{sigma}, b]$   
 quaternion algebra  $(a, b)_K$        $B = K, C = [a, b]$   
 matrix algebra  $M_d(K)$        $B = K, C = d$   
 local Hasse invariants over  $K$        $B = K, C = [d, [PR, HF], HI]$

## Properties

type of  $al$  ( $mt$ , CSA)      **algtype**( $al$ )  
 is  $al$  a division algebra? (at place  $v$ )      **algisdivision**( $al, \{v\}$ )  
 dimension of  $al$  over its center      **algdim**( $al$ )  
 degree of  $A$  ( $= \sqrt{\dim}$ )      **algdegree**( $al$ )  
 index of  $A$  over  $K$  (index at  $v$ )      **algindex**( $al, \{v\}$ )  
 $al$  a cyclic algebra  $(L/K, \sigma, b)$ ; return  $\sigma$       **algaut**( $al$ )  
 $\dots$  return  $b$       **algb**( $al$ )  
 $\dots$  return  $L/K$ , as an  $rnf$       **algsplittingfield**( $al$ )  
 split  $A$  over an extension of  $K$       **algsplittingdata**( $al$ )  
 splitting field of  $A$  as an  $rnf$  over center      **algsplittingfield**( $al$ )  
 places of  $K$  at which  $A$  ramifies      **algramifiedplaces**( $al$ )  
 Hasse invariants at finite places of  $K$       **alghassef**( $al$ )  
 Hasse invariants at infinite places of  $K$       **alghassei**( $al$ )  
 Hasse invariant at place  $v$       **alghasse**( $al, v$ )

## Operations on elements

reduced norm      **algnorm**( $al, x$ )  
 reduced trace      **algtrace**( $al, x$ )  
 reduced char. polynomial      **algcharpoly**( $al, x$ )  
 express  $x$  on integral basis      **algalgtobasis**( $al, x$ )  
 convert  $x$  to algebraic form      **algbasistoalg**( $al, x$ )  
 map  $x \in A$  to  $M_d(L)$ ,  $L$  split. field      **algsplittingmatrix**( $al, x$ )

## Orders

$\mathbf{Z}$ -basis of order  $\mathcal{O}_0$       **algbasis**( $al$ )  
 discriminant of order  $\mathcal{O}_0$       **algdisc**( $al$ )  
 $\mathbf{Z}$ -basis of natural order in terms  $\mathcal{O}_0$ 's basis      **alginvbasis**( $al$ )

Based on an earlier version by Joseph H. Silverman

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