PARI-GP Reference Card

(PARI-GP version 2.6.1)

Note: optional arguments are surrounded by braces {}.
To start the calculator, type its name in the terminal: `gp`
To exit `gp`, type `quit`, `\q`, or `<C-D>` at prompt.

Help

describe function	`?`*function*
extended description	`??`*keyword*
list of relevant help topics	`???`*pattern*

Input/Output

previous result, the result before	`%`, `` %` ``, ``` %`` ```, etc.
n-th result since startup	`%`n
separate multiple statements on line	`;`
extend statement on additional lines	`\`
extend statements on several lines	`{`seq_1`;` seq_2`;}`
comment	`/* ... */`
one-line comment, rest of line ignored	`\\ ...`

Metacommands & Defaults

set default d to *val*	`default`$(\{d\}, \{val\}, \{flag\})$
toggle timer on/off	`#`
print time for last result	`##`
print defaults	`\d`
set debug level to n	`\g` n
set memory debug level to n	`\gm` n
set output mode (raw=0, default=1)	`\o` n
set n significant digits	`\p` n
set n terms in series	`\ps` n
quit GP	`\q`
print the list of PARI types	`\t`
print the list of user-defined functions	`\u`
read file into GP	`\r` *filename*

Debugger / break loop

get out of break loop	`break` or `<C-D>`
go up n frames	`dbg_up`$(\{n\})$
examine object o	`dbg_x`(o)

PARI Types & Input Formats

`t_INT`/`t_REAL`. Integers, Reals	$\pm n$, $\pm n.ddd$
`t_INTMOD`. Integers modulo m	`Mod`(n, m)
`t_FRAC`. Rational Numbers	n/m
`t_FFELT`. Elt in finite field F_q	`ffgen(q)`
`t_COMPLEX`. Complex Numbers	$x + y *$ `I`
`t_PADIC`. p-adic Numbers	$x +$ `O`$(p$`^`$k)$
`t_QUAD`. Quadratic Numbers	$x + y *$ `quadgen`(D)
`t_POLMOD`. Polynomials modulo g	`Mod`(f, g)
`t_POL`. Polynomials	$a * x$`^`$n + \cdots + b$
`t_SER`. Power Series	$f +$ `O`$(x$`^`$k)$
`t_QFI`/`t_QFR`. Imag/Real bin. quad. forms	`Qfb`$(a, b, c, \{d\})$
`t_RFRAC`. Rational Functions	f/g
`t_VEC`/`t_COL`. Row/Column Vectors	$[x, y, z]$, $[x, y, z]$`~`
`t_MAT`. Matrices	$[x, y; z, t; u, v]$
`t_LIST`. Lists	`List`$([x, y, z])$
`t_STR`. Strings	`"abc"`

Reserved Variable Names

$\pi = 3.14\ldots$, $\gamma = 0.57\ldots$, $C = 0.91\ldots$	`Pi`, `Euler`, `Catalan`
square root of -1	`I`
big-oh notation	`O`

Information about an Object

PARI type of object x	`type`(x)
length of x / size of x in memory	`#`x, `sizebyte`(x)
real or p-adic precision of x	`precision`(x), `padicprec`

Operators

basic operations	`+`, `-` , `*`, `/`, `^`
`i=i+1, i=i-1, i=i*j, ...`	`i++, i--, i*=j,...`
euclidean quotient, remainder	x`\/`y, x`\`y, x`%`y, `divrem`(x, y)
shift x left or right n bits	x`<<`n, x`>>`n or `shift`$(x, \pm n)$
comparison operators	`<=`, `<`, `>=`, `>`, `==`, `!=`, `===`, `lex`, `cmp`
boolean operators (or, and, not)	`||`, `&&`, `!`
bit operations	`bitand`, `bitneg`, `bitor`, `bitxor`
sign of $x = -1, 0, 1$	`sign`(x)
maximum/minimum of x and y	`max`, `min`(x, y)
integer or real factorial of x	x`!` or `factorial`(x)
derivative of f w.r.t. x	f`'`
apply differential operator	`diffop`
restore x as a formal variable	x`='`x
simultaneous assignment $x \leftarrow v_1$, $y \leftarrow v_2$	`[x,y] = v`

Select Components

n-th component of x	`component`(x, n)
n-th component of vector/list x	$x[n]$
components $a, a+1, \ldots, b$ of vector x	$x[a..b]$
(m, n)-th component of matrix x	$x[m, n]$
row m or column n of matrix x	$x[m,]$, $x[,n]$
numerator/denominator of x	`numerator`(x), `denominator`

Conversions

to vector, matrix, set, list, string	`Col`/`Vec`,`Mat`,`Set`,`List`,`Str`
create PARI object (x mod y)	`Mod`(x, y)
make x a polynomial of v	`Pol`$(x, \{v\})$
as `Pol`/`Vec`, starting with constant term	`Polrev`, `Vecrev`
make x a power series of v	`Ser`$(x, \{v\})$
string from bytes / from format+args	`Strchr`, `Strprintf`
convert x to simplest possible type	`simplify`(x)
object x with precision n	`precision`(x, n)

Conjugates and Lifts

conjugate of a number x	`conj`(x)
conjugate vector of algebraic number x	`conjvec`(x)
norm of x, product with conjugate	`norm`(x)
square of L^2 norm of vector x	`norml2`(x)
lift of x from Mods	`lift`, `centerlift`(x)

Lists, Sets & Sorting

sort x by k-th component	`vecsort`$(x, \{k\}, \{fl = 0\})$
min. m of x ($m = x[i]$), max.	`vecmin`$(x, \{\&i\})$, `vecmax`
does y belong to x, sorted wrt. f	`vecsearch`$(x, y, \{f\})$

Sets (= row vector of strings with strictly increasing entries)

intersection of sets x and y	`setintersect`(x, y)
set of elements in x not belonging to y	`setminus`(x, y)
union of sets x and y	`setunion`(x, y)
does y belong to the set x	`setsearch`$(x, y, \{flag\})$
is x a set ?	`setisset`(x)

Lists. create empty list: $L = \text{List}()$

append x to list L	`listput`$(L, x, \{i\})$
remove i-th component from list L	`listpop`$(L, \{i\})$
insert x in list L at position i	`listinsert`(L, x, i)
sort the list L in place	`listsort`$(L, \{flag\})$

Programming

Functions and closures

`fun(vars) = my(local vars);` *seq*
`fun = (vars) -> my(local vars);` *seq*

Control Statements (X: formal parameter in expression *seq*)

eval. *seq* for $a \le X \le b$	`for`$(X = a, b, seq)$
eval. *seq* for X dividing n	`fordiv`(n, X, seq)
eval. *seq* for primes $a \le X \le b$	`forprime`$(X = a, b, seq)$
eval. *seq* for $a \le X \le b$ stepping s	`forstep`$(X = a, b, s, seq)$
multivariable `for`	`forvec`$(X = v, seq)$
loop over partitions of n	`forpart`$(p = n\ seq)$
loop over vectors v, $q(v) \le B$, $q > 0$	`forqfvec`(v, q, b, seq)
loop over subgrps H of abelian grp G	`forsubgroup`$(H = G)$
evaluate *seq* until $a \ne 0$	`until`(a, seq)
while $a \ne 0$, evaluate *seq*	`while`(a, seq)
exit n innermost enclosing loops	`break`$(\{n\})$
start new iteration of n-th enclosing loop	`next`$(\{n\})$
return x from current subroutine	`return`$(\{x\})$
raise an exception	`error()`
if $a \ne 0$, evaluate seq_1, else seq_2	`if`$(a, \{seq_1\}, \{seq_2\})$
try seq_1, evaluate seq_2 on error	`iferr`(seq_1, E, seq_2)
select from v according to f	`select`(f, v)
apply f to all entries in v	`apply`(f, v)

Input/Output

print with/without `\n`, TeX format	`print`, `print1`, `printtex`
formatted printing	`printf()`
write *args* to file	`write`, `write1`, `writetex`$(file, args)$
write x in binary format	`writebin`$(file, x)$
read file into GP	`read`$(\{file\})$
read file, return as vector of lines	`readvec`$(\{file\})$
read a string from keyboard	`input()`

Interface with User and System

allocates a new stack of s bytes	`allocatemem`$(\{s\})$
alias *old* to *new*	`alias`(new, old)
install function from library	`install`$(f, code, \{gpf\}, \{lib\})$
execute system command a	`system`(a)
as above, feed result to GP	`extern`(a)
as above, return GP string	`externstr`(a)
get `$VAR` from environment	`getenv("VAR")`
measure time in ms.	`gettime()`
timeout command after s seconds	`alarm`$(s, expr)$

Iterations, Sums & Products

numerical integration	`intnum`$(X = a, b, expr, \{flag\})$
sum *expr* over divisors of n	`sumdiv`$(n, X, expr)$
`sumdiv`, with *expr* multiplicative	`sumdivmult`$(n, X, expr)$
sum $X = a$ to $X = b$, initialized at x	`sum`$(X = a, b, expr, \{x\})$
sum of series *expr*	`suminf`$(X = a, expr)$
sum of alternating/positive series	`sumalt`, `sumpos`
sum of series using `intnum`	`sumnum`
product $a \le X \le b$, initialized at x	`prod`$(X = a, b, expr, \{x\})$
product over primes $a \le X \le b$	`prodeuler`$(X = a, b, expr)$
infinite product $a \le X \le \infty$	`prodinf`$(X = a, expr)$
real root of *expr* between a and b	`solve`$(X = a, b, expr)$

Random Numbers

random integer/prime in $[0, N[$	`random`(N), `randomprime`
get/set random seed	`getrand`, `setrand`(s)

Vectors & Matrices

dimensions of matrix x	`matsize(x)`
concatenation of x and y	`concat(x, {y})`
extract components of x	`vecextract(x, y, {z})`
transpose of vector or matrix x	`mattranspose(x)` or `x~`
adjoint of the matrix x	`matadjoint(x)`
eigenvectors/values of matrix x	`mateigen(x)`
characteristic/minimal polynomial of x	`charpoly(x)`, `minpoly`
trace/determinant of matrix x	`trace(x)`, `matdet`
Frobenius form of x	`matfrobenius(x)`
QR decomposition	`matqr(x)`

Constructors & Special Matrices

row vec. of *expr* eval'ed at $1 \leq i \leq n$	`vector(n, {i}, {expr})`
col. vec. of *expr* eval'ed at $1 \leq i \leq n$	`vectorv(n, {i}, {expr})`
matrix $1 \leq i \leq m$, $1 \leq j \leq n$	`matrix(m, n, {i}, {j}, {expr})`
define matrix by blocks	`matconcat(B)`
diagonal matrix with diagonal x	`matdiagonal(x)`
$n \times n$ identity matrix	`matid(n)`
Hessenberg form of square matrix x	`mathess(x)`
$n \times n$ Hilbert matrix $H_{ij} = (i+j-1)^{-1}$	`mathilbert(n)`
companion matrix to polynomial x	`matcompanion(x)`
Sylvester matrix of x	`polsylvestermatrix(x)`

Gaussian elimination

kernel of matrix x	`matker(x, {flag})`
intersection of column spaces of x and y	`matintersect(x, y)`
solve $M * X = B$ (M invertible)	`matsolve(M, B)`
as solve, modulo D (col. vector)	`matsolvemod(M, D, B)`
one sol of $M * X = B$	`matinverseimage(M, B)`
basis for image of matrix x	`matimage(x)`
supplement columns of x to get basis	`matsupplement(x)`
rows, cols to extract invertible matrix	`matindexrank(x)`
rank of the matrix x	`matrank(x)`

Lattices & Quadratic Forms

upper triangular Hermite Normal Form	`mathnf(x)`
HNF of x where d is a multiple of $\det(x)$	`mathnfmod(x, d)`
elementary divisors of x	`matsnf(x)`
LLL-algorithm applied to columns of x	`qflll(x, {flag})`
like `qflll`, x is Gram matrix of lattice	`qflllgram(x, {flag})`
LLL-reduced basis for kernel of x	`matkerint(x)`
$\mathbf{Z}$-lattice $\longleftrightarrow$ $\mathbf{Q}$-vector space	`matrixqz(x, p)`
signature of quad form ${}^t y * x * y$	`qfsign(x)`
decomp into squares of ${}^t y * x * y$	`qfgaussred(x)`
eigenvals/eigenvecs for real symmetric x	`qfjacobi(x)`
find up to m sols of ${}^t y * x * y \leq b$	`qfminim(x, b, m)`
perfection rank of x	`qfperfection(x)`
v, $v[i]$:=number of sols of ${}^t y * x * y = i$	`qfrep(x, B, {flag})`
automorphism group of q	`qfauto(q)`
find isomorphism between q and Q	`qfisom(q, Q)`

Formal & p-adic Series

truncate power series or p-adic number	`truncate(x)`
valuation of x at p	`valuation(x, p)`

Dirichlet and Power Series

Taylor expansion around 0 of f w.r.t. x	`taylor(f, x)`
$\sum a_k b_k t^k$ from $\sum a_k t^k$ and $\sum b_k t^k$	`serconvol(a, b)`
$f = \sum a_k t^k$ from $\sum (a_k / k!) t^k$	`serlaplace(f)`
reverse power series F so $F(f(x)) = x$	`serreverse(f)`
Dirichlet series multiplication / division	`dirmul`, `dirdiv(x, y)`
Dirichlet Euler product (b terms)	`direuler(p=a, b, expr)`

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Polynomials & Rational Functions

degree of f	`poldegree(f)`
coeff. of degree n of f, leading coeff.	`polcoeff(f, n)`, `pollead`
gcd of coefficients of f	`content(f)`
replace x by y	`subst(f, x, y)`
evaluate f replacing vars by their value	`eval(f)`
replace polynomial expr. $T(x)$ by y in f	`substpol(f, T, y)`
replace $x_1, \ldots, x_n$ by $y_1, \ldots, y_n$ in f	`substvec(f, x, y)`
discriminant of polynomial f	`poldisc(f)`
resultant $R = \mathrm{Res}_v(f, g)$	`polresultant(f, g, {v})`
$[u, v, R]$, $xu + yv = \mathrm{Res}_v(f, g)$	`polresultantext(x, y, {v})`
derivative of f w.r.t. x	`deriv(f, {x})`
formal integral of f w.r.t. x	`intformal(f, {x})`
formal sum of f w.r.t. x	`sumformal(f, {x})`
reciprocal poly $x^{\deg f} f(1/x)$	`polrecip(f)`
interpol. pol. eval. at a	`polinterpolate(X, {Y}, {a}, {&e})`
initialize t for Thue equation solver	`thueinit(f)`
solve Thue equation $f(x, y) = a$	`thue(t, a, {sol})`

Roots and Factorization

number of real roots of f, $a < x \leq b$	`polsturm(f, {a}, {b})`
complex roots of f	`polroots(f)`
symmetric powers of roots of f up to n	`polsym(f, n)`
factor f	`factor(f, {lim})`
factor f mod p / roots	`factormod(f, p)`, `polrootsmod`
factor f over $\mathbf{F}_{p^a}$ / roots	`factorff(f, p, a)`, `polrootsff`
factor f over $\mathbf{Q}_p$ / roots	`factorpadic(f, p, r)`, `polrootspadic`
find irreducible $T \in \mathbf{F}_p[x]$, $\deg T = n$	`ffinit(p, n, {x})`
$\#\{$monic irred. $T \in \mathbf{F}_q[x], \deg T = n\}$	`ffnbirred(q, n)`
p-adic root of f cong. to a mod p	`padicappr(f, a)`
Newton polygon of f for prime p	`newtonpoly(f, p)`
extensions of $\mathbf{Q}_p$ of degree N	`padicfields(p, N)`

Special Polynomials

n-th cyclotomic polynomial in var. v	`polcyclo(n, {v})`
d-th degree subfield of $\mathbf{Q}(\zeta_n)$	`polsubcyclo(n, d, {v})`
P_n, T_n/U_n, H_n	`pollegendre`, `polchebyshev`, `polhermite`

Transcendental and p-adic Functions

real, imaginary part of x	`real(x)`, `imag(x)`
absolute value, argument of x	`abs(x)`, `arg(x)`
square/nth root of x	`sqrt(x)`, `sqrtn(x, n, {&z})`
trig functions	`sin`, `cos`, `tan`, `cotan`
inverse trig functions	`asin`, `acos`, `atan`
hyperbolic functions	`sinh`, `cosh`, `tanh`
inverse hyperbolic functions	`asinh`, `acosh`, `atanh`
exponential / natural log of x	`exp`, `log`
Euler Γ function, $\log \Gamma$, Γ'/Γ	`gamma`, `lngamma`, `psi`
incomplete gamma function ($y = \Gamma(s)$)	`incgam(s, x, {y})`
exponential integral $\int_x^\infty e^{-t}/t \, dt$	`eint1(x)`
error function $2/\sqrt{\pi} \int_x^\infty e^{-t^2} dt$	`erfc(x)`
dilogarithm of x	`dilog(x)`
m-th polylogarithm of x	`polylog(m, x, {flag})`
U-confluent hypergeometric function	`hyperu(a, b, u)`
Bessel $J_n(x)$, $J_{n+1/2}(x)$	`besselj(n, x)`, `besseljh(n, x)`
Bessel I_ν, K_ν, H^1_ν, H^2_ν, N_ν	`(bessel)i`, `k`, `h1`, `h2`, `n`
Lambert W: x s.t. $xe^x = y$	`lambertw(y)`
Teichmuller character of p-adic x	`teichmuller(x)`

Elementary Arithmetic Functions

vector of binary digits of $\lvert x \rvert$	`binary(x)`
bit number n of integer x	`bittest(x, n)`
Hamming weight of integer x	`hammingweight(x)`
ceiling/floor/fractional part	`ceil`, `floor`, `frac`
round x to nearest integer	`round(x, {&e})`
truncate x	`truncate(x, {&e})`
gcd/LCM of x and y	`gcd(x, y)`, `lcm(x, y)`
gcd of entries of a vector/matrix	`content(x)`

Primes and Factorization

add primes in v to prime table	`addprimes(v)`
Chebyshev $\pi(x)$, n-th prime p_n	`primepi(x)`, `prime(n)`
vector of first n primes	`primes(n)`
smallest prime $\geq x$	`nextprime(x)`
largest prime $\leq x$	`precprime(x)`
factorization of x	`factor(x, {lim})`
$n = df^2$, d squarefree/fundamental	`core(n, {fl})`, `coredisc`
recover x from its factorization	`factorback(f, {e})`

Divisors

number of prime divisors $\omega(n)$ / $\Omega(n)$	`omega(n)`, `bigomega`
divisors of n / number of divisors $\tau(n)$	`divisors(n)`, `numdiv`
sum of (k-th powers of) divisors of n	`sigma(n, {k})`

Special Functions and Numbers

binomial coefficient $\binom{x}{y}$	`binomial(x, y)`
Bernoulli number B_n as real/rational	`bernreal(n)`, `bernfrac`
Bernoulli polynomial $B_n(x)$	`bernpol(n, {x})`
n-th Fibonacci number	`fibonacci(n)`
Stirling numbers $s(n, k)$ and $S(n, k)$	`stirling(n, k, {flag})`
number of partitions of n	`numbpart(n)`
Möbius μ-function	`moebius(x)`
Hilbert symbol of x and y (at p)	`hilbert(x, y, {p})`
Kronecker-Legendre symbol $(\frac{x}{y})$	`kronecker(x, y)`
Dedekind sum $s(h, k)$	`sumdedekind(h, k)`

Multiplicative groups $(\mathbf{Z}/N\mathbf{Z})^*$, $\mathbf{F}_q^*$

Euler ϕ-function	`eulerphi(x)`
multiplicative order of x (divides o)	`znorder(x, {o})`, `fforder`
primitive root mod q / `x.mod`	`znprimroot(q)`, `ffprimroot(x)`
structure of $(\mathbf{Z}/n\mathbf{Z})^*$	`znstar(n)`
discrete logarithm of x in base g	`znlog(x, g, {o})`, `fflog`

Miscellaneous

integer square / n-th root of x	`sqrtint(x)`, `sqrtnint(x, n)`
solve $z \equiv x$ and $z \equiv y$	`chinese(x, y)`
minimal u, v so $xu + yv = \gcd(x, y)$	`gcdext(x, y)`
continued fraction of x	`contfrac(x, {b}, {lmax})`
last convergent of continued fraction x	`contfracpnqn(x)`
rational approximation to x	`bestappr(x, k)`, `bestapprPade`

True-False Tests

is x the disc. of a quadratic field?	`isfundamental(x)`
is x a prime?	`isprime(x)`
is x a strong pseudo-prime?	`ispseudoprime(x)`
is x square-free?	`issquarefree(x)`
is x a square?	`issquare(x, {&n})`
is x a perfect power?	`ispower(x, {k}, {&n})`
is *pol* irreducible?	`polisirreducible(pol)`

Send comments and corrections to ⟨Karim.Belabas@math.u-bordeaux.fr⟩

PARI-GP Reference Card (2)

(PARI-GP version 2.6.1)

Elliptic Curves

Elliptic curve initially given by 5-tuple $v =$[a_1, a_2, a_3, a_4, a_6].

Initialize *ell* struct	E = ellinit(v, {*Domain*})

Points are [x,y], the origin is [0]. Struct members accessed as E.*member*:

- All domains: E.a1,a2,a3,a4,a6, b2,b4,b6,b8, c4,c6, disc, j
- E defined over $\mathbf{R}$ or $\mathbf{C}$

x-coords. of points of order 2	E.roots
periods / quasi-periods	E.omega,E.eta
volume of complex lattice	E.area

- E defined over $\mathbf{Q}_p$

residual characteristic	E.p
If $\lvert j\rvert_p > 1$: Tate's $[u^2, u, q, [a, b]]$	E.tate

- E defined over $\mathbf{F}_q$

characteristic	E.p
$\#E(\mathbf{F}_q)$/cyclic structure/generators	E.no, E.cyc, E.gen

- E defined over $\mathbf{Q}$

generators of $E(\mathbf{Q})$ (require elldata)	E.gen
$[a_1, a_2, a_3, a_4, a_6]$ from j-invariant	ellfromj(j)
change curve E using $v =$[u, r, s, t]	ellchangecurve(E, v)
change point z using $v =$[u, r, s, t]	ellchangepoint(z, v)
add points $P + Q$ / $P - Q$	elladd(E, P, Q), ellsub
negate point	ellneg(E, P)
compute $n \cdot z$	ellmul(E, z, n)
n-division polynomial $f_n(x)$	elldivpol(E, n, {x})
check if z is on E	ellisoncurve(E, z)
order of torsion point z	ellorder(E, z)
y-coordinates of point(s) for x	ellordinate(E, x)
point $[\wp(z), \wp'(z)]$ corresp. to z	ellztopoint(E, z)
complex z such that $p = [\wp(z), \wp'(z)]$	ellpointtoz(E, p)

Curves over finite fields, Pairings

random point on E	random(E)
$\#E(\mathbf{F}_q)$	ellcard(E)
structure $\mathbf{Z}/d_1\mathbf{Z} \times \mathbf{Z}/d_2\mathbf{Z}$ of $E(\mathbf{F}_q)$	ellgroup(E)
Weil pairing of m-torsion pts x, y	ellweilpairing(E, x, y, m)
Tate pairing of x, y; x m-torsion	elltatepairing(E, x, y, m)
Discrete log, find n s.t. $P = [n]Q$	elllog(E, P, Q, {*ord*})

Curves over Q and the L-function

canonical bilinear form taken at z_1, z_2	ellbil(E, z_1, z_2)
canonical height of z	ellheight(E, z, {*flag*})
height regulator matrix for pts in x	ellheightmatrix(E, x)
cond, min mod, Tamagawa num [N, v, c]	ellglobalred(E)
reduction of $y^2 + Qy = P$ (genus 2)	genus2red(Q, P, {p})
Kodaira type of p-fiber of E	elllocalred(E, p)
minimal model of $E/\mathbf{Q}$	ellminimalmodel(E, {&v})
p-th coeff a_p of L-function, p prime	ellap(E, p)
k-th coeff a_k of L-function	ellak(E, k)
vector of first n a_k's in L-function	ellan(E, n)
$L(E, s)$	elllseries(E, s)
$L^{(r)}(E, 1)$	ellL1(E, r)
return a Heegner point on E of rank 1	ellheegner(E)
order of vanishing at 1	ellanalyticrank(E, {*eps*})
root number for $L(E, .)$ at p	ellrootno(E, {p})
torsion subgroup with generators	elltors(E)
modular parametrization of E	elltaniyama(E)

Elldata package, Cremona's database:

db code $\leftrightarrow$ [*conductor*, *class*, *index*]	ellconvertname(s)
generators of Mordell-Weil group	ellgenerators(E)
look up E in database	ellidentify(E)
all curves matching criterion	ellsearch(N)
loop over curves with cond. from a to b	forell(E, a, b, seq)

Elliptic & Modular Functions

$w = [\omega_1, \omega_2]$ or *ell* struct (E.omega), $\tau = \omega_1/\omega_2$.

arithmetic-geometric mean	agm(x, y)
elliptic j-function $1/q + 744 + \cdots$	ellj(x)
Weierstrass $\sigma/\wp/\zeta$ function	ellsigma(w, z), ellwp, ellzeta
periods/quasi-periods	ellperiods(E, {*flag*}), elleta(w)
$(2i\pi/\omega_2)^k E_k(\tau)$	elleisnum(w, k, {*flag*})
modified Dedekind η func. $\prod(1 - q^n)$	eta(x, {*flag*})
Jacobi sine theta function	theta(q, z)
k-th derivative at z=0 of theta(q, z)	thetanullk(q, k)
Weber's f functions	weber(x, {*flag*})
Riemann's zeta $\zeta(s) = \sum n^{-s}$	zeta(s)

Binary Quadratic Forms

create $ax^2 + bxy + cy^2$ (distance d)	Qfb(a, b, c, {d})
reduce x ($s = \sqrt{D}$, $l = \lfloor s\rfloor$)	qfbred(x, {*flag*}, {D}, {l}, {s})
composition of forms	x*y or qfbnucomp(x, y, l)
n-th power of form	x^n or qfbnupow(x, n)
composition without reduction	qfbcompraw(x, y)
n-th power without reduction	qfbpowraw(x, n)
prime form of disc. x above prime p	qfbprimeform(x, p)
class number of disc. x	qfbclassno(x)
Hurwitz class number of disc. x	qfbhclassno(x)
Solve $Q(x, y) = p$ in integers, p prime	qfbsolve(Q, p)

Quadratic Fields

quadratic number $\omega = \sqrt{x}$ or $(1 + \sqrt{x})/2$	quadgen(x)
minimal polynomial of ω	quadpoly(x)
discriminant of $\mathbf{Q}(\sqrt{D})$	quaddisc(x)
regulator of real quadratic field	quadregulator(x)
fundamental unit in real $\mathbf{Q}(x)$	quadunit(x)
class group of $\mathbf{Q}(\sqrt{D})$	quadclassunit(D, {*flag*}, {t})
Hilbert class field of $\mathbf{Q}(\sqrt{D})$	quadhilbert(D, {*flag*})
ray class field modulo f of $\mathbf{Q}(\sqrt{D})$	quadray(D, f, {*flag*})

General Number Fields: Initializations

A number field K is given by a monic irreducible $f \in \mathbf{Z}[X]$.

init number field structure *nf*	nfinit(f, {*flag*})

nf members:

polynomial defining *nf*, $f(\theta) = 0$	*nf*.pol
number of real/complex places	*nf*.r1/r2/sign
discriminant of *nf*	*nf*.disc
T_2 matrix	*nf*.t2
vector of roots of f	*nf*.roots
integral basis of $\mathbf{Z}_K$ as powers of θ	*nf*.zk
different	*nf*.diff
codifferent	*nf*.codiff
index	*nf*.index
recompute *nf* using current precision	nfnewprec(*nf*)
init relative *rnf* given by $g = 0$ over K	rnfinit(*nf*, g)
init*bnf* structure	bnfinit(f, {*flag*})

bnf members: same as *nf*, plus

underlying *nf*	*bnf*.nf
classgroup	*bnf*.clgp
regulator	*bnf*.reg
fundamental units	*bnf*.fu
torsion units	*bnf*.tu
compute a *bnf* from small *bnf*	bnfinit(*sbnf*)
add S-class group and units, yield *bnf* s	bnfsunit(*nf*, S)
init class field structure *bnr*	bnrinit(*bnf*, m, {*flag*})

bnr members: same as *bnf*, plus

underlying *bnf*	*bnr*.bnf
big ideal structure	*bnr*.bid
modulus	*bnr*.mod
structure of $(\mathbf{Z}_K/m)^*$	*bnr*.zkst

Basic Number Field Arithmetic (nf)

Elements are t_INT, t_FRAC, t_POL, t_POLMOD, or t_COL (on integral basis *nf*.zk). Basic operations (prefix nfelt): (nfelt)add, mul, pow, div, diveuc, mod, divrem, val, trace, norm

express x on integer basis	nfalgtobasis(*nf*, x)
express element x as a polmod	nfbasistoalg(*nf*, x)
reverse polmod $a = A(X)$ mod $T(X)$	modreverse(a)
integral basis of field def. by $f = 0$	nfbasis(f)
field discriminant of field $f = 0$	nfdisc(f)
smallest poly defining $f = 0$ (slow)	polredabs(f, {*flag*})
small poly defining $f = 0$ (fast)	polredbest(f, {*flag*})
are fields $f = 0$ and $g = 0$ isomorphic?	nfisisom(f, g)
is field $f = 0$ a subfield of $g = 0$?	nfisincl(f, g)
compositum of $f = 0$, $g = 0$	polcompositum(f, g, {*flag*})
subfields (of degree d) of *nf*	nfsubfields(*nf*, {d})
roots of unity in *nf*	nfrootsof1(*nf*)
roots of g belonging to *nf*	nfroots({*nf*}, g)
factor g in *nf*	nffactor(*nf*, g)
factor g mod prime *pr* in *nf*	nffactormod(*nf*, g, *pr*)
conjugates of a root θ of *nf*	nfgaloisconj(*nf*, {*flag*})
apply Galois automorphism s to x	nfgaloisapply(*nf*, s, x)
quadratic Hilbert symbol (at p)	nfhilbert(*nf*, a, b, {p})

Linear and algebraic relations

poly of degree $\le k$ with root $x \in \mathbf{C}$	algdep(x, k)
alg. dep. with pol. coeffs for series s	seralgdep(s, x, y)
small linear rel. on coords of vector x	lindep(x)

Dedekind Zeta Function ζ_K, Hecke L series

ζ_K as Dirichlet series, $N(I) < b$	dirzetak(*nf*, b)
init *nfz* for field $f = 0$	zetakinit(f)
compute $\zeta_K(s)$	zetak(*nfz*, s, {*flag*})
Artin root number of K	bnrrootnumber(*bnr*, *chi*, {*flag*})
$L(1, \chi)$, for all χ trivial on H	bnrL1(*bnr*, {H}, {*flag*})

Class Groups & Units (bnf, bnr)

a_1, {a_2}, {a_3} usually *bnr*, *subgp* or *bnf*, *module*, {*subgp*}

remove GRH assumption from *bnf*	bnfcertify(*bnf*)
expo. of ideal x on class gp	bnfisprincipal(*bnf*, x, {*flag*})
expo. of ideal x on ray class gp	bnrisprincipal(*bnr*, x, {*flag*})
expo. of x on fund. units	bnfisunit(*bnf*, x)
as above for S-units	bnfissunit(*bnfs*, x)
signs of real embeddings of *bnf*.fu	bnfsignunit(*bnf*)
narrow class group	bnfnarrow(*bnf*)

PARI-GP Reference Card (2)

(PARI-GP version 2.6.1)

Class Field Theory

ray class number for mod. m	`bnrclassno(bnf,m)`
discriminant of class field ext	`bnrdisc(a1,{a2},{a3})`
ray class numbers, l list of mods	`bnrclassnolist(bnf,l)`
discriminants of class fields	`bnrdisclist(bnf,l,{arch},{flag})`
decode output from `bnrdisclist`	`bnfdecodemodule(nf,fa)`
is modulus the conductor?	`bnrisconductor(a1,{a2},{a3})`
conductor of character *chi*	`bnrconductorofchar(bnr,chi)`
conductor of extension	`bnrconductor(a1,{a2},{a3},{flag})`
conductor of extension def. by g	`rnfconductor(bnf,g)`
Artin group of ext. def'd by g	`rnfnormgroup(bnr,g)`
subgroups of *bnr*, index $<= b$	`subgrouplist(bnr,b,{flag})`
rel. eq. for class field def'd by *sub*	`rnfkummer(bnr,sub,{d})`
same, using Stark units (real field)	`bnrstark(bnr,sub,{flag})`

Ideals: elements, primes, or matrix of generators in HNF

is *id* an ideal in *nf* ?	`nfisideal(nf,id)`
is x principal in *bnf* ?	`bnfisprincipal(bnf,x)`
give $[a,b]$, s.t. $a\mathbf{Z}_K + b\mathbf{Z}_K = x$	`idealtwoelt(nf,x,{a})`
put ideal a ($a\mathbf{Z}_K + b\mathbf{Z}_K$) in HNF form	`idealhnf(nf,a,{b})`
norm of ideal x	`idealnorm(nf,x)`
minimum of ideal x (direction v)	`idealmin(nf,x,v)`
LLL-reduce the ideal x (direction v)	`idealred(nf,x,{v})`

Ideal Operations

add ideals x and y	`idealadd(nf,x,y)`
multiply ideals x and y	`idealmul(nf,x,y,{flag})`
intersection of ideals x and y	`idealintersect(nf,x,y,{flag})`
n-th power of ideal x	`idealpow(nf,x,n,{flag})`
inverse of ideal x	`idealinv(nf,x)`
divide ideal x by y	`idealdiv(nf,x,y,{flag})`
Find $(a,b) \in x \times y$, $a+b=1$	`idealaddtoone(nf,x,{y})`
coprime integral A, B such that $x = A/B$	`idealnumden(nf,x)`

Primes and Multiplicative Structure

factor ideal x in *nf*	`idealfactor(nf,x)`
expand ideal factorization in *nf*	`idealfactorback(nf,f,e)`
decomposition of prime p in *nf*	`idealprimedec(nf,p)`
valuation of x at prime ideal *pr*	`idealval(nf,x,pr)`
weak approximation theorem in *nf*	`idealchinese(nf,x,y)`
give *bid* =structure of $(\mathbf{Z}_K/id)^*$	`idealstar(nf,id,{flag})`
discrete log of x in $(\mathbf{Z}_K/bid)^*$	`ideallog(nf,x,bid)`
`idealstar` of all ideals of norm $\leq b$	`ideallist(nf,b,{flag})`
add Archimedean places	`ideallistarch(nf,b,{ar},{flag})`
init `prmod` structure	`nfmodprinit(nf,pr)`
kernel of matrix M in $(\mathbf{Z}_K/pr)^*$	`nfkermodpr(nf,M,prmod)`
solve $Mx = B$ in $(\mathbf{Z}_K/pr)^*$	`nfsolvemodpr(nf,M,B,prmod)`

Galois theory over $\mathbf{Q}$

Galois group of field $\mathbf{Q}[x]/(f)$	`polgalois(f)`
initializes a Galois group structure G	`galoisinit(pol,{den})`
action of p in nfgaloisconj form	`galoispermtopol(G,{p})`
identify as abstract group	`galoisidentify(G)`
export a group for GAP/MAGMA	`galoisexport(G,{flag})`
subgroups of the Galois group G	`galoissubgroups(G)`
is subgroup H normal?	`galoisisnormal(G,H)`
subfields from subgroups	`galoissubfields(G,{flag},{v})`
fixed field	`galoisfixedfield(G,perm,{flag},{v})`
Frobenius at maximal ideal P	`idealfrobenius(nf,G,P)`
ramification groups at P	`idealramgroups(nf,G,P)`
is G abelian?	`galoisisabelian(G,{flag})`
abelian number fields/$\mathbf{Q}$	`galoissubcyclo(N,H,{flag},{v})`
query the `galpol` package	`galoisgetpol(a,b,{s})`

Relative Number Fields (rnf)

Extension L/K is defined by $T \in K[x]$.

absolute equation of L	`rnfequation(nf,T,{flag})`
is L/K abelian?	`rnfisabelian(nf,T)`
relative `nfalgtobasis`	`rnfalgtobasis(rnf,x)`
relative `nfbasistoalg`	`rnfbasistoalg(rnf,x)`
relative `idealhnf`	`rnfidealhnf(rnf,x)`
relative `idealmul`	`rnfidealmul(rnf,x,y)`
relative `idealtwoelt`	`rnfidealtwoelt(rnf,x)`

Lifts and Push-downs

absolute → relative repres. for x	`rnfeltabstorel(rnf,x)`
relative → absolute repres. for x	`rnfeltreltoabs(rnf,x)`
lift x to the relative field	`rnfeltup(rnf,x)`
push x down to the base field	`rnfeltdown(rnf,x)`

idem for x ideal: `(rnfideal)reltoabs`, `abstorel`, `up`, `down`

Norms

absolute norm of ideal x	`rnfidealnormabs(rnf,x)`
relative norm of ideal x	`rnfidealnormrel(rnf,x)`
solutions of $N_{K/\mathbf{Q}}(y) = x \in \mathbf{Z}$	`bnfisintnorm(bnf,x)`
is $x \in \mathbf{Q}$ a norm from K?	`bnfisnorm(bnf,x,{flag})`
initialize T for norm eq. solver	`rnfisnorminit(K,pol,{flag})`
is $a \in K$ a norm from L?	`rnfisnorm(T,a,{flag})`

Maximal order $\mathbf{Z}_L$ as a $\mathbf{Z}_K$-module

relative `polred`	`rnfpolred(nf,T)`
relative `polredabs`	`rnfpolredabs(nf,T)`
characteristic poly. of a mod T	`rnfcharpoly(nf,T,a,{v})`
relative Dedekind criterion, prime *pr*	`rnfdedekind(nf,T,pr)`
discriminant of relative extension	`rnfdisc(nf,T)`
pseudo-basis of $\mathbf{Z}_L$	`rnfpseudobasis(nf,T)`

General $\mathbf{Z}_K$-modules: $M = [\text{matrix}, \text{vec. of ideals}] \subset L$

relative HNF / SNF	`nfhnf(nf,M)`, `nfsnf`
reduced basis for M	`rnflllgram(nf,T,M)`
determinant of pseudo-matrix M	`rnfdet(nf,M)`
Steinitz class of M	`rnfsteinitz(nf,M)`
$\mathbf{Z}_K$-basis of M if $\mathbf{Z}_K$-free, or 0	`rnfhnfbasis(bnf,M)`
n-basis of M, or $(n+1)$-generating set	`rnfbasis(bnf,M)`
is M a free $\mathbf{Z}_K$-module?	`rnfisfree(bnf,M)`

Graphic Functions

crude graph of *expr* between a and b	`plot(X=a,b,expr)`

High-resolution plot (immediate plot)

plot *expr* between a and b	`ploth(X=a,b,expr,{flag},{n})`
plot points given by lists lx, ly	`plothraw(lx,ly,{flag})`
terminal dimensions	`plothsizes()`

Rectwindow functions

init window w, with size x,y	`plotinit(w,x,y)`
erase window w	`plotkill(w)`
copy w to w_2 with offset (dx,dy)	`plotcopy(w,w2,dx,dy)`
clips contents of w	`plotclip(w)`
scale coordinates in w	`plotscale(w,x1,x2,y1,y2)`
`ploth` in w	`plotrecth(w,X=a,b,expr,{flag},{n})`
`plothraw` in w	`plotrecthraw(w,data,{flag})`
draw window w_1 at (x_1,y_1), ...	`plotdraw([[w1,x1,y1],...])`

Low-level Rectwindow Functions

set current drawing color in w to c	`plotcolor(w,c)`
current position of cursor in w	`plotcursor(w)`
write s at cursor's position	`plotstring(w,s)`
move cursor to (x,y)	`plotmove(w,x,y)`
move cursor to $(x+dx, y+dy)$	`plotrmove(w,dx,dy)`
draw a box to (x_2,y_2)	`plotbox(w,x2,y2)`
draw a box to $(x+dx, y+dy)$	`plotrbox(w,dx,dy)`
draw polygon	`plotlines(w,lx,ly,{flag})`
draw points	`plotpoints(w,lx,ly)`
draw line to $(x+dx, y+dy)$	`plotrline(w,dx,dy)`
draw point $(x+dx, y+dy)$	`plotrpoint(w,dx,dy)`
draw point $(x+dx, y+dy)$	`plotrpoint(w,dx,dy)`

Postscript Functions

as `ploth`	`psploth(X=a,b,expr,{flag},{n})`
as `plothraw`	`psplothraw(lx,ly,{flag})`
as `plotdraw`	`psdraw([[w1,x1,y1],...])`

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