

Pari-GP reference card

(PARI-GP version 2.10.0)

Note: optional arguments are surrounded by braces {}.

To start the calculator, type its name in the terminal: **gp**

To exit **gp**, type **quit**, **\q**, or **<C-D>** at prompt.

Help

describe function	?function
extended description	??keyword
list of relevant help topics	???pattern
name of GP-1.39 function f in GP-2.*	whatnow(f)

Input/Output

previous result, the result before	%, %', %'', etc.
n -th result since startup	% n
separate multiple statements on line	;
extend statement on additional lines	\
extend statements on several lines	{seq ₁ ; seq ₂ ;}
comment	/* ... */
one-line comment, rest of line ignored	\\ ...

Metacommands & Defaults

set default d to val	default({ d },{ val })
toggle timer on/off	#
print time for last result	##
print defaults	\d
set debug level to n	\g n
set memory debug level to n	\gm n
set n significant digits / bits	\p n , \pb n
set n terms in series	\ps n
quit GP	\q
print the list of PARI types	\t
print the list of user-defined functions	\u
read file into GP	\r filename

Debugger / break loop

get out of break loop	break or <C-D>
go up/down n frames	dbg_up({ n }), dbg_down
set break point	breakpoint()
examine object o	dbg_x(o)
current error data	dbg_err()
number of objects on heap and their size	getheap()
total size of objects on PARI stack	getstack()

PARI Types & Input Formats

t_INT . Integers; hex, binary	± 31 ; $\pm 0x1F$, $\pm 0b101$
t_REAL . Reals	± 3.14 , $6.022\ E23$
t_INTMOD . Integers modulo m	Mod(n , m)
t_FRAC . Rational Numbers	n/m
t_FFELT . Elt in finite field F_q	ffgen(q)
t_COMPLEX . Complex Numbers	$x + y * I$
t_PADIC . p -adic Numbers	$x + O(p^k)$
t_QUAD . Quadratic Numbers	$x + y * \text{quadgen}(D)$
t_POLMOD . Polynomials modulo g	Mod(f , g)
t_POL . Polynomials	$a * x^n + \dots + b$
t_SER . Power Series	$f + O(x^k)$
t_RFRAC . Rational Functions	f/g
t_QFI / t_QFR . Imag/Real binary quad. form	Qfb($a, b, c, \{d\}$)
t_VEC / t_COL . Row/Column Vectors	[x, y, z], [x, y, z]-
t_VEC integer range	[1..10]

t_VECSMALL . Vector of small ints	Vecsmall([x, y, z])
t_MAT . Matrices	[$a, b; c, d$]
t_LIST . Lists	List([x, y, z])
t_STR . Strings	"abc"
t_INFINITY . $\pm\infty$	+oo, -oo

Reserved Variable Names

$\pi = 3.14\dots$, $\gamma = 0.57\dots$, $C = 0.91\dots$	Pi, Euler, Catalan
square root of -1	I
Landau's big-oh notation	O

Information about an Object

PARI type of object x	type(x)
length of x / size of x in memory	# x , sizebyte(x)
real precision / bit precision of x	precision(x), bitprecision
p -adic, series prec. of x	padicprec(x), serprec

Operators

basic operations	+, -, *, /, ^, sqr
$i=i+1$, $i=i-1$, $i=i*j$, ...	i++, i--, i*=j,...
euclidean quotient, remainder	$x \backslash y$, $x \backslash y$, $x \% y$, divrem(x, y)
shift x left or right n bits	$x << n$, $x >> n$ or shift($x, \pm n$)
multiply by 2^n	shiftmul(x, n)
comparison operators	<=, <, >=, >, ==, !=, ==, lex, cmp
boolean operators (or, and, not)	, &&, !
bit operations	bitand, bitneg, bitor, bitxor, bitnegimply
sign of $x = -1, 0, 1$	sign(x)
maximum/minimum of x and y	max, min(x, y)
derivative of f	f'
differential operator	diffop($f, v, d, \{n = 1\}$)
quote operator (formal variable)	' x
assignment	$x = value$
simultaneous assignment $x \leftarrow v_1, y \leftarrow v_2$	[x, y] = v

Select Components

n -th component of x	component(x, n)
n -th component of vector/list x	$x[n]$
components $a, a + 1, \dots, b$ of vector x	$x[a..b]$
(m, n) -th component of matrix x	$x[m, n]$
row m or column n of matrix x	$x[m,]$, $x[, n]$
numerator/denominator of x	numerator(x), denominator

Random Numbers

random integer/prime in $[0, N[$	random(N), randomprime
get/set random seed	getrand, setrand(s)

Conversions

to vector, matrix, vec. of small ints	Col/Vec, Mat, Vecsmall
to list, set, map, string	List, Set, Map, Str
create PARI object ($x \bmod y$)	Mod(x, y)
make x a polynomial of v	Pol($x, \{v\}$)
as Pol, etc., starting with constant term	Polrev, Vecrev, Colrev
make x a power series of v	Ser($x, \{v\}$)
string from bytes / from format+args	Strchr, Strprintf
TeX string	Strtex(x)
convert x to simplest possible type	simplify(x)
object x with real precision n	precision(x, n)
object x with bit precision n	bitprecision(x, n)
set precision to p digits in dynamic scope	localprec(p)
set precision to p bits in dynamic scope	localbitprec(p)

Conjugates and Lifts

conjugate of a number x	conj(x)
norm of x , product with conjugate	norm(x)
L^p norm of x (L^∞ if no p)	normlp($x, \{p\}$)
square of L^2 norm of x	norml2(x)
lift of x from Mods and p -adics	lift, centerlift(x)
recursive lift	liftall
lift all t_INT and t_PADIC (\rightarrow t_INT)	liftint
lift all t_POLMOD (\rightarrow t_POL)	liftpol

Lists, Sets & Maps

Sets (= row vector with strictly increasing entries w.r.t. cmp)	
intersection of sets x and y	setintersect(x, y)
set of elements in x not belonging to y	setminus(x, y)
union of sets x and y	setunion(x, y)
does y belong to the set x	setsearch($x, y, \{flag\}$)
set of all $f(x, y)$, $x \in X$, $y \in Y$	setbinop(f, X, Y)
is x a set ?	setisset(x)

Lists . create empty list: $L = \text{List}()$	
append x to list L	listput($L, x, \{i\}$)
remove i -th component from list L	listpop($L, \{i\}$)
insert x in list L at position i	listinsert(L, x, i)
sort the list L in place	listsort($L, \{flag\}$)

Maps . create empty dictionary: $M = \text{Map}()$	
attach value v to key k	mapput(M, k, v)
recover value attach to key k or error	mapget(M, k)
is key k in the dict ? (set v to $M(k)$)	mapisdefined($M, k, \{\&v\}$)
remove k from map domain	mapdelete(M, k)

GP Programming

User functions and closures

x, y are formal parameters; y defaults to Pi if parameter opitted; z, t are local variables (lexical scope), z initialized to 1.

fun (x , $y=\text{Pi}$) = my ($z=1$, t); seq	
fun = (x , $y=\text{Pi}$) -> my ($z=1$, t); seq	
attach a help message to f	addhelp(f)
undefine symbol s (also kills help)	kill(s)
Control Statements (X : formal parameter in expression seq)	
if $a \neq 0$, evaluate seq_1 , else seq_2	if($a, \{seq_1\}, \{seq_2\}$)
eval. seq for $a \leq X \leq b$	for($X = a, b, seq$)
...for primes $a \leq X \leq b$	forprime($X = a, b, seq$)
...for composites $a \leq X \leq b$	forcomposite($X = a, b, seq$)
...for $a \leq X \leq b$ stepping s	forstep($X = a, b, s, seq$)
...for X dividing n	fordiv(n, X, seq)
multivariable for , lex ordering	forvec($X = v, seq$)
loop over partitions of n	forpart($p = n, seq$)
loop over vectors v , $q(v) \leq B$; $q > 0$	forqfvec(v, q, b, seq)
loop over $H < G$ finite abelian group	forsubgroup($H = G$)

evaluate seq until $a \neq 0$	until(a, seq)
while $a \neq 0$, evaluate seq	while(a, seq)
exit n innermost enclosing loops	break({ n })
start new iteration of n -th enclosing loop	next({ n })
return x from current subroutine	return({ x })
Exceptions, warnings	
raise an exception / warn	error(), warning()
type of error message E	errname(E)
try seq_1 , evaluate seq_2 on error	iferr(seq_1, E, seq_2)

Functions with closure arguments / results

select from v according to f	<code>select(f, v)</code>
apply f to all entries in v	<code>apply(f, v)</code>
evaluate $f(a_1, \dots, a_n)$	<code>call(f, a)</code>
evaluate $f(\dots f(f(a_1, a_2), a_3) \dots, a_n)$	<code>fold(f, a)</code>
calling function as closure	<code>self()</code>

Sums & Products

sum $X = a$ to $X = b$, initialized at x	<code>sum($X = a, b, expr, \{x\}$)</code>
sum entries of vector v	<code>vecsum(v)</code>
sum $expr$ over divisors of n	<code>sumdiv($n, X, expr$)</code>
... assuming $expr$ multiplicative	<code>sumdivmult($n, X, expr$)</code>
product $a \leq X \leq b$, initialized at x	<code>prod($X = a, b, expr, \{x\}$)</code>
product over primes $a \leq X \leq b$	<code>prodeuler($X = a, b, expr$)</code>

Sorting

sort x by k -th component	<code>vecsort($x, \{k\}, \{fl = 0\}$)</code>
min. m of x ($m = x[i]$), max.	<code>vecmin($x, \{\&i\}$), <code>vecmax</code></code>
does y belong to x , sorted wrt. f	<code>vecsearch($x, y, \{f\}$)</code>

Input/Output

print with/without $\backslash n$, \TeX format	<code>print, print1, printtex</code>
print fields with separator	<code>printsep(sep, \dots), <code>printsep1</code></code>
formatted printing	<code>printf()</code>
write $args$ to file	<code>write, write1, writetex($file, args$)</code>
write x in binary format	<code>writebin($file, x$)</code>
read file into GP	<code>read($\{file\}$)</code>
... return as vector of lines	<code>readvec($\{file\}$)</code>
... return as vector of strings	<code>readstr($\{file\}$)</code>
read a string from keyboard	<code>input()</code>

Timers

CPU time in ms and reset timer	<code>gettime()</code>
CPU time in ms since gp startup	<code>getabstime()</code>
time in ms since UNIX Epoch	<code>getwalltime()</code>
timeout command after s seconds	<code>alarm($s, expr$)</code>

Interface with system

allocates a new stack of s bytes	<code>allocatemem($\{s\}$)</code>
alias old to new	<code>alias(new, old)</code>
install function from library	<code>install($f, code, \{gpf\}, \{lib\}$)</code>
execute system command a	<code>system(a)</code>
as above, feed result to GP	<code>extern(a)</code>
as above, return GP string	<code>externstr(a)</code>
get $\$VAR$ from environment	<code>getenv("VAR")</code>
expand env. variable in string	<code>Strexpand(x)</code>

Parallel evaluation

These functions evaluate their arguments in parallel (pthreads or MPI); args. must not access global variables and must be free of side effects. Enabled if threading engine is not <i>single</i> in gp header.	
evaluate f on $x[1], \dots, x[n]$	<code>parapply(f, x)</code>
evaluate closures $f[1], \dots, f[n]$	<code>pareval(f)</code>
as <code>select</code>	<code>parselect($f, A, \{flag\}$)</code>
as <code>sum</code>	<code>parsum($i = a, b, expr, \{x\}$)</code>
as <code>vector</code>	<code>parvector($n, i, \{expr\}$)</code>
eval f for $i = a, \dots, b$	<code>parfor($i = a, \{b\}, f, \{r\}, \{f_2\}$)</code>
... for p prime in $[a, b]$	<code>parforprime($p = a, \{b\}, f, \{r\}, \{f_2\}$)</code>
... multivariate	<code>parforvec($X = v, f, \{r\}, \{f_2\}, \{flag\}$)</code>
declare x as inline (allows to use as global)	<code>inline(x)</code>
stop inlining	<code>uninline()</code>

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Linear Algebra

dimensions of matrix x	<code>matsize(x)</code>
concatenation of x and y	<code>concat($x, \{y\}$)</code>
extract components of x	<code>vecextract($x, y, \{z\}$)</code>
transpose of vector or matrix x	<code>mattranspose(x)</code> or $x\sim$
adjoint of the matrix x	<code>matadjoint(x)</code>
eigenvectors/values of matrix x	<code>mateigen(x)</code>
characteristic/minimal polynomial of x	<code>charpoly(x), <code>minpoly</code></code>
trace/determinant of matrix x	<code>trace(x), <code>matdet</code></code>
Frobenius form of x	<code>matfrobenius(x)</code>
QR decomposition	<code>matqr(x)</code>
apply <code>matqr</code> 's transform to v	<code>mathouseholder(Q, v)</code>

Constructors & Special Matrices

$\{g(x): x \in v \text{ s.t. } f(x)\}$	<code>[$g(x) \mid x \leftarrow v, f(x)$]</code>
$\{x: x \in v \text{ s.t. } f(x)\}$	<code>[$x \mid x \leftarrow v, f(x)$]</code>
$\{g(x): x \in v\}$	<code>[$g(x) \mid x \leftarrow v$]</code>
row vec. of $expr$ eval'ed at $1 \leq i \leq n$	<code>vector($n, \{i\}, \{expr\}$)</code>
col. vec. of $expr$ eval'ed at $1 \leq i \leq n$	<code>vectorv($n, \{i\}, \{expr\}$)</code>
vector of small ints	<code>vectorsmall($n, \{i\}, \{expr\}$)</code>
$[c, c \cdot x, \dots, c \cdot x^n]$	<code>powers($x, n, \{c = 1\}$)</code>
matrix $1 \leq i \leq m, 1 \leq j \leq n$	<code>matrix($m, n, \{i\}, \{j\}, \{expr\}$)</code>
define matrix by blocks	<code>matconcat(B)</code>
diagonal matrix with diagonal x	<code>matdiagonal(x)</code>
is x diagonal?	<code>matisdiagonal(x)</code>
$x \cdot \text{matdiagonal}(d)$	<code>matmuldiagonal(x, d)</code>
$n \times n$ identity matrix	<code>matid(n)</code>
Hessenberg form of square matrix x	<code>mathess(x)</code>
$n \times n$ Hilbert matrix $H_{ij} = (i + j - 1)^{-1}$	<code>mathilbert(n)</code>
$n \times n$ Pascal triangle	<code>matpascal($n - 1$)</code>
companion matrix to polynomial x	<code>matcompanion(x)</code>
Sylvester matrix of x	<code>polsylvestermatrix(x)</code>

Gaussian elimination

kernel of matrix x	<code>matker($x, \{flag\}$)</code>
intersection of column spaces of x and y	<code>matintersect(x, y)</code>
solve $M * X = B$ (M invertible)	<code>matsolve(M, B)</code>
as solve, modulo D (col. vector)	<code>matsolvemod(M, D, B)</code>
one sol of $M * X = B$	<code>matinverseimage(M, B)</code>
basis for image of matrix x	<code>matimage(x)</code>
columns of x not in <code>matimage</code>	<code>matimagecompl(x)</code>
supplement columns of x to get basis	<code>matsupplement(x)</code>
rows, cols to extract invertible matrix	<code>matindexrank(x)</code>
rank of the matrix x	<code>matrank(x)</code>

Lattices & Quadratic Forms

Quadratic forms

evaluate ${}^t x Q y$	<code>qfeval($\{Q = id\}, x, y$)</code>
evaluate ${}^t x Q x$	<code>qfeval($\{Q = id\}, x$)</code>
signature of quad form ${}^t y * x * y$	<code>qfsign(x)</code>
decomp into squares of ${}^t y * x * y$	<code>qfgaussred(x)</code>
eigenvalues/vectors for real symmetric x	<code>qfjacobi(x)</code>

HNF and SNF

upper triangular Hermite Normal Form	<code>mathnf(x)</code>
HNF of x where d is a multiple of $\det(x)$	<code>mathnfmod(x, d)</code>
multiple of $\det(x)$	<code>matdetint(x)</code>
HNF of $(x \mid \text{diagonal}(D))$	<code>mathnfmodid(x, D)</code>
elementary divisors of x	<code>mathsnf(x)</code>
elementary divisors of $\mathbf{Z}[a]/(f'(a))$	<code>poldiscreduced(f)</code>
integer kernel of x	<code>matkerint(x)</code>
\mathbf{Z} -module \leftrightarrow \mathbf{Q} -vector space	<code>matrixqz(x, p)</code>

Lattices

LLL-algorithm applied to columns of x	<code>qflll($x, \{flag\}$)</code>
... for Gram matrix of lattice	<code>qflllgram($x, \{flag\}$)</code>
find up to m sols of <code>qfnorm</code> ($x, y\}) \leq b$	<code>qfminim(x, b, m)</code>
$v, v[i] :=$ number of y s.t. <code>qfnorm</code> ($x, y\}) = i$	<code>qfrep($x, B, \{flag\}$)</code>
perfection rank of x	<code>qfperfection(x)</code>
find isomorphism between q and Q	<code>qfiso(q, Q)</code>
precompute for isomorphism test with q	<code>qfisoinit(q)</code>
automorphism group of q	<code>qfauto(q)</code>
convert <code>qfauto</code> for GAP/Magma	<code>qfautoexport($G, \{flag\}$)</code>
orbits of V under $G \subset \text{GL}(V)$	<code>qforbits(G, V)</code>

Polynomials & Rational Functions

all defined polynomial variables	<code>variables()</code>
get var. of highest priority (higher than v)	<code>varhigher($name, \{v\}$)</code>
... of lowest priority (lower than v)	<code>varlower($name, \{v\}$)</code>
Coefficients, variables and basic operators	
degree of f	<code>poldegree(f)</code>
coeff. of degree n of f , leading coeff.	<code>polcoeff(f, n), <code>pollead</code></code>
main variable / all variables in f	<code>variable(f), <code>variables(f)</code></code>
replace x by y in f	<code>subst(f, x, y)</code>
evaluate f replacing vars by their value	<code>eval(f)</code>
replace polynomial expr. $T(x)$ by y in f	<code>substpol(f, T, y)</code>
replace x_1, \dots, x_n by y_1, \dots, y_n in f	<code>substvec(f, x, y)</code>
reciprocal polynomial $x^{\deg f} f(1/x)$	<code>polrecip(f)</code>
gcd of coefficients of f	<code>content(f)</code>
derivative of f w.r.t. x	<code>deriv($f, \{x\}$)</code>
formal integral of f w.r.t. x	<code>intformal($f, \{x\}$)</code>
formal sum of f w.r.t. x	<code>sumformal($f, \{x\}$)</code>

Constructors & Special Polynomials

interpolating pol. eval. at a	<code>polinterpolate($X, \{Y\}, \{a\}$)</code>
$P_n, T_n/U_n, H_n$	<code>pollegendre, polchebyshev, polhermite</code>
n -th cyclotomic polynomial Φ_n	<code>polcyclo($n, \{v\}$)</code>
return n if $f = \Phi_n$, else 0	<code>poliscyclo(f)</code>
is f a product of cyclotomic polynomials?	<code>poliscycloprod(f)</code>
Zagier's polynomial of index (n, m)	<code>polzagier(n, m)</code>

Resultant, elimination

discriminant of polynomial f	<code>poldisc(f)</code>
resultant $R = \text{Res}_v(f, g)$	<code>polresultant($f, g, \{v\}$)</code>
$[u, v, R], xu + yv = \text{Res}_v(f, g)$	<code>polresultantext($x, y, \{v\}$)</code>
solve Thue equation $f(x, y) = a$	<code>thue($t, a, \{sol\}$)</code>
initialize t for Thue equation solver	<code>thueinit(f)</code>

Based on an earlier version by Joseph H. Silverman

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Roots and Factorization

complex roots of f	<code>polroots(f)</code>
number of real roots of f (in $[a, b]$)	<code>polsturm($f, \{[a, b]\}$)</code>
real roots of f (in $[a, b]$)	<code>polrootsreal($f, \{[a, b]\}$)</code>
symmetric powers of roots of f up to n	<code>polsym(f, n)</code>
Graeffe transform of f , $g(x^2) = f(x)f(-x)$	<code>polgraeffe(f)</code>
factor f	<code>factor(f)</code>
factor f mod p / roots	<code>factormod(f, p), polrootsmod</code>
... using Cantor-Zassenhaus	<code>factorcantor(f, p)</code>
factor f over \mathbf{F}_{p^a} / roots	<code>factorff(f, p, a), polrootsff</code>
factor f over \mathbf{Q}_p / roots	<code>factorpadic(f, p, r), polrootspadic</code>
cyclotomic factors of $f \in \mathbf{Q}[X]$	<code>polcyclofactors(f)</code>
find irreducible $T \in \mathbf{F}_p[x]$, $\deg T = n$	<code>ffinit($p, n, \{x\}$)</code>
$\#\{\text{monic irred. } T \in \mathbf{F}_q[x], \deg T = n\}$	<code>ffnbirred(q, n)</code>
p -adic root of f congruent to a mod p	<code>padicappr(f, a)</code>
Newton polygon of f for prime p	<code>newtonpoly(f, p)</code>
Hensel lift $A/\text{lc}(A) = \prod_i B[i] \bmod p^e$	<code>polhensellift(A, B, p, e)</code>
extensions of \mathbf{Q}_p of degree N	<code>padicfields(p, N)</code>

Formal & p-adic Series

truncate power series or p -adic number	<code>truncate(x)</code>
valuation of x at p	<code>valuation(x, p)</code>
Dirichlet and Power Series	
Taylor expansion around 0 of f w.r.t. x	<code>taylor(f, x)</code>
$\sum a_k b_k t^k$ from $\sum a_k t^k$ and $\sum b_k t^k$	<code>serconvol(a, b)</code>
$f = \sum a_k t^k$ from $\sum (a_k/k!) t^k$	<code>serlaplace(f)</code>
reverse power series F so $F(f(x)) = x$	<code>serreverse(f)</code>
Dirichlet series multiplication / division	<code>dirmul, dirdiv(x, y)</code>
Dirichlet Euler product (b terms)	<code>direuler($p = a, b, expr$)</code>

Transcendental and p -adic Functions

real, imaginary part of x	<code>real(x), imag(x)</code>
absolute value, argument of x	<code>abs(x), arg(x)</code>
square/nth root of x	<code>sqrt(x), sqrtn($x, n, \{&z\}$)</code>
trig functions	<code>sin, cos, tan, cotan, sinc</code>
inverse trig functions	<code>asin, acos, atan</code>
hyperbolic functions	<code>sinh, cosh, tanh, cotanh</code>
inverse hyperbolic functions	<code>asinh, acosh, atanh</code>
$\log(x)$, e^x , $e^x - 1$	<code>log, exp, expm1</code>
Euler Γ function, $\log \Gamma$, Γ'/Γ	<code>gamma, lngamma, psi</code>
half-integer gamma function $\Gamma(n+1/2)$	<code>gammah(n)</code>
Riemann's zeta $\zeta(s) = \sum n^{-s}$	<code>zeta(s)</code>
multiple zeta value (MZV), $\zeta(s_1, \dots, s_k)$	<code>zetamult(s)</code>
incomplete Γ function ($y = \Gamma(s)$)	<code>incgam($s, x, \{y\}$)</code>
complementary incomplete Γ	<code>incgamc(s, x)</code>
exponential integral $\int_x^\infty e^{-t}/t dt$	<code>eint1(x)</code>
error function $2/\sqrt{\pi} \int_x^\infty e^{-t^2} dt$	<code>erfc(x)</code>
dilogarithm of x	<code>dilog(x)</code>
m -th polylogarithm of x	<code>polylog($m, x, \{flag\}$)</code>
U -confluent hypergeometric function	<code>hyperu(a, b, u)</code>
Bessel $J_n(x)$, $J_{n+1/2}(x)$	<code>besselj(n, x), besseljh(n, x)</code>
Bessel I_ν , K_ν , H_ν^1 , H_ν^2 , N_ν	<code>(bessel)i, k, h1, h2, n</code>
Lambert W : x s.t. $xe^x = y$	<code>lambertw(y)</code>
Teichmuller character of p -adic x	<code>teichmuller(x)</code>

Iterations, Sums & Products

Numerical integration for meromorphic functions

Behaviour at endpoint for Double Exponential methods: either a scalar ($a \in \mathbf{C}$, regular) or $\pm\infty$ (decreasing at least as x^{-2}) or	
$(x-a)^{-\alpha}$ singularity	<code>[a, α]</code>
exponential decrease $e^{-\alpha x }$	<code>[$\pm\infty, \alpha$], $\alpha > 0$</code>
slow decrease $ x ^\alpha$	<code>... $\alpha < -1$</code>
oscillating as $\cos(kx)$	<code>$\alpha = k\mathbf{I}$, $k > 0$</code>
oscillating as $\sin(kx)$	<code>$\alpha = -k\mathbf{I}$, $k > 0$</code>
numerical integration	<code>intnum($x = a, b, f, \{T\}$)</code>
weights T for intnum	<code>intnuminit($a, b, \{m\}$)</code>
weights T incl. kernel K	<code>intfuncinit($a, b, K, \{m\}$)</code>
integrate $(2i\pi)^{-1}f$ on circle $ z-a = R$	<code>intcirc($x = a, R, f, \{T\}$)</code>

Other integration methods

n -point Gauss-Legendre	<code>intnumgauss($x = a, b, f, \{n\}$)</code>
weights for n -point Gauss-Legendre	<code>intnumgaussinit($\{n\}$)</code>
Romberg integration (low accuracy)	<code>intnumromb($x = a, b, f, \{flag\}$)</code>

Numerical summation

sum of series $f(n)$, $n \geq a$ (low accuracy)	<code>suminf($n = a, expr$)</code>
sum of alternating/positive series	<code>sumalt, sumpos</code>
sum of series using Euler-Maclaurin	<code>sumnum($n = a, f, \{T\}$)</code>
weights for sumnum, a as in DE	<code>sumnuminit($\{\infty, a\}$)</code>
sum of series by Monien summation	<code>sumnummonien($n = a, f, \{T\}$)</code>
weights for sumnummonien	<code>sumnummonieninit($\{\infty, a\}$)</code>

Products

product $a \leq X \leq b$, initialized at x	<code>prod($X = a, b, expr, \{x\}$)</code>
product over primes $a \leq X \leq b$	<code>prodeuler($X = a, b, expr$)</code>
infinite product $a \leq X \leq \infty$	<code>prodinf($X = a, expr$)</code>

Other numerical methods

real root of f in $[a, b]$; bracketed root	<code>solve($X = a, b, f$)</code>
... by interval splitting	<code>solvestep($X = a, b, f, \{flag = 0\}$)</code>
limit of $f(t)$, $t \rightarrow \infty$	<code>limitnum($f, \{k\}, \{\alpha\}$)</code>
asymptotic expansion of f at ∞	<code>asymnum($f, \{k\}, \{\alpha\}$)</code>
numerical derivation w.r.t x : $f'(a)$	<code>derivnum($x = a, f$)</code>
evaluate continued fraction F at t	<code>contfraceval($F, t, \{L\}$)</code>
power series to cont. fraction (L terms)	<code>contfracinit($S, \{L\}$)</code>
Padé approximant (deg. denom. $\leq B$)	<code>bestapprPade($S, \{B\}$)</code>

Elementary Arithmetic Functions

vector of binary digits of $ x $	<code>binary(x)</code>
bit number n of integer x	<code>bittest(x, n)</code>
Hamming weight of integer x	<code>hammingweight(x)</code>
digits of integer x in base B	<code>digits($x, \{B = 10\}$)</code>
sum of digits of integer x in base B	<code>sumdigits($x, \{B = 10\}$)</code>
integer from digits	<code>fromdigits($v, \{B = 10\}$)</code>
ceiling/floor/fractional part	<code>ceil, floor, frac</code>
round x to nearest integer	<code>round($x, \{&e\}$)</code>
truncate x	<code>truncate($x, \{&e\}$)</code>
gcd/LCM of x and y	<code>gcd(x, y), lcm(x, y)</code>
gcd of entries of a vector/matrix	<code>content(x)</code>

Primes and Factorization

extra prime table	<code>addprimes()</code>
add primes in v to prime table	<code>addprimes(v)</code>
remove primes from prime table	<code>removeprimes(v)</code>
Chebyshev $\pi(x)$, n -th prime p_n	<code>primepi(x), prime(n)</code>
vector of first n primes	<code>primes(n)</code>
smallest prime $\geq x$	<code>nextprime(x)</code>
largest prime $\leq x$	<code>precprime(x)</code>
factorization of x	<code>factor($x, \{lim\}$)</code>
... selecting specific algorithms	<code>factorint($x, \{flag = 0\}$)</code>
$n = df^2$, d squarefree/fundamental	<code>core($n, \{fl\}$), coredisc</code>
recover x from its factorization	<code>factorback($f, \{e\}$)</code>
$x \in \mathbf{Z}$, $ x \leq X$, $\gcd(N, P(x)) \geq N$	<code>zncoppersmith($P, N, X, \{B\}$)</code>

Divisors and multiplicative functions

number of prime divisors $\omega(n)$ / $\Omega(n)$	<code>omega(n), bigomega</code>
divisors of n / number of divisors $\tau(n)$	<code>divisors(n), numdiv</code>
sum of (k -th powers of) divisors of n	<code>sigma($n, \{k\}$)</code>
Möbius μ -function	<code>moebius(x)</code>
Ramanujan's τ -function	<code>ramanujantau(x)</code>

Combinatorics

factorial of x	<code>$x!$ or factorial(x)</code>
binomial coefficient $\binom{x}{y}$	<code>binomial(x, y)</code>
Bernoulli number B_n as real/rational	<code>bernreal(n), bernfrac</code>
Bernoulli polynomial $B_n(x)$	<code>bernpol($n, \{x\}$)</code>
n -th Fibonacci number	<code>fibonacci(n)</code>
Stirling numbers $s(n, k)$ and $S(n, k)$	<code>stirling($n, k, \{flag\}$)</code>
number of partitions of n	<code>numbpart(n)</code>
k -th permutation on n letters	<code>numtoperm(n, k)</code>
convert permutation to (n, k) form	<code>permtonum(v)</code>

Multiplicative groups $(\mathbf{Z}/N\mathbf{Z})^*$, \mathbf{F}_q^*

Euler ϕ -function	<code>eulerphi(x)</code>
multiplicative order of x (divides ϕ)	<code>znorder($x, \{o\}$), fforder</code>
primitive root mod q / $x \bmod q$	<code>znprimroot(q), fprimroot(x)</code>
structure of $(\mathbf{Z}/n\mathbf{Z})^*$	<code>znstar(n)</code>
discrete logarithm of x in base g	<code>znlog($x, g, \{o\}$), fflog</code>
Kronecker-Legendre symbol $\left(\frac{x}{y}\right)$	<code>kronecker(x, y)</code>
quadratic Hilbert symbol (at p)	<code>hilbert($x, y, \{p\}$)</code>

Miscellaneous

integer square / n -th root of x	<code>sqrtint(x), sqrtntint(x, n)</code>
largest integer e s.t. $b^e \leq b$, $e = \lfloor \log_b(b) \rfloor$	<code>logint($x, b, \{&z\}$)</code>
CRT: solve $z \equiv x$ and $z \equiv y$	<code>chinese(x, y)</code>
minimal u, v so $ux + yv = \gcd(x, y)$	<code>gcdext(x, y)</code>
continued fraction of x	<code>contfrac($x, \{b\}, \{lmax\}$)</code>
last convergent of continued fraction x	<code>contfracpnqn(x)</code>
rational approximation to x (den. $\leq B$)	<code>bestappr($x, \{B\}k$)</code>

Characters

Let $cyc = [d_1, \dots, d_k]$ represent an abelian group $G = \oplus (\mathbf{Z}/d_j\mathbf{Z}) \cdot g_j$ or any structure G affording a `.cyc` method; e.g. `idealstar(,g)` for Dirichlet characters. A character χ is coded by $[c_1, \dots, c_k]$ such that $\chi(g_j) = e(n_j/d_j)$.

$\chi \cdot \psi$; χ^{-1} ; $\chi \cdot \psi^{-1}$	<code>charmul, charconj, chardiv</code>
order of χ	<code>charorder(cyc, χ)</code>
kernel of χ	<code>charker(cyc, χ)</code>
$\chi(x)$, G a GP group structure	<code>chareval($G, \chi, x, \{z\}$)</code>

Dirichlet Characters

initialize $G = (\mathbf{Z}/q\mathbf{Z})^*$ $G = \text{idealstar}(,q)$
is χ odd? $\text{zncharisodd}(G,\chi)$
real $\chi \rightarrow$ Kronecker symbol $(D/.)$ $\text{znchartokronecker}(G,\chi)$
induce $\chi \in \hat{G}$ to $\mathbf{Z}/N\mathbf{Z}$ $\text{zncharinduce}(G,chi,N)$

Conrey labelling

Conrey label $m \in (\mathbf{Z}/q\mathbf{Z})^* \rightarrow$ character $\text{znconreychar}(G,m)$
character \rightarrow Conrey label $\text{znconreyexp}(G,\chi)$
log on Conrey generators $\text{znconreylog}(G,m)$
conductor of χ (χ_0 primitive) $\text{znconreyconductor}(G,\chi,\{\chi_0\})$

True-False Tests

is x the disc. of a quadratic field? $\text{isfundamental}(x)$
is x a prime? $\text{isprime}(x)$
is x a strong pseudo-prime? $\text{ispseudoprime}(x)$
is x square-free? $\text{issquarefree}(x)$
is x a square? $\text{issquare}(x,\{\&n\})$
is x a perfect power? $\text{ispower}(x,\{k\},\{\&n\})$
is x a perfect power of a prime? ($x = p^n$) $\text{isprimepower}(x,\&n\})$
... of a pseudoprime? $\text{ispseudoprimepower}(x,\&n\})$
is x powerful? $\text{ispowerful}(x)$
is x a totient? ($x = \varphi(n)$) $\text{istotient}(x,\{\&n\})$
is x a polygonal number? ($x = P(s,n)$) $\text{ispolygonal}(x,s,\{\&n\})$
is pol irreducible? $\text{polisirreducible}(pol)$

Graphic Functions

crude graph of $expr$ between a and b $\text{plot}(X = a,b,expr)$
High-resolution plot (immediate plot)
plot $expr$ between a and b $\text{ploth}(X = a,b,expr,\{flag\},\{n\})$
plot points given by lists lx, ly $\text{plothraw}(lx,ly,\{flag\})$
terminal dimensions $\text{plotsizes}()$

Rectwindow functions

init window w , with size x,y $\text{plotinit}(w,x,y)$
erase window w $\text{plotkill}(w)$
copy w to w_2 with offset (dx,dy) $\text{plotcopy}(w,w_2,dx,dy)$
clips contents of w $\text{plotclip}(w)$
scale coordinates in w $\text{plotscale}(w,x_1,x_2,y_1,y_2)$
 ploth in w $\text{plotrecth}(w,X = a,b,expr,\{flag\},\{n\})$
 plothraw in w $\text{plotrecthraw}(w,data,\{flag\})$
draw window w_1 at $(x_1,y_1), \dots$ $\text{plotdraw}([[w_1,x_1,y_1],\dots])$

Low-level Rectwindow Functions

set current drawing color in w to c $\text{plotcolor}(w,c)$
current position of cursor in w $\text{plotcursor}(w)$
write s at cursor's position $\text{plotstring}(w,s)$
move cursor to (x,y) $\text{plotmove}(w,x,y)$
move cursor to $(x+dx,y+dy)$ $\text{plotrmove}(w,dx,dy)$
draw a box to (x_2,y_2) $\text{plotbox}(w,x_2,y_2)$
draw a box to $(x+dx,y+dy)$ $\text{plotrbox}(w,dx,dy)$
draw polygon $\text{plotlines}(w,lx,ly,\{flag\})$
draw points $\text{plotpoints}(w,lx,ly)$
draw line to $(x+dx,y+dy)$ $\text{plotrline}(w,dx,dy)$
draw point $(x+dx,y+dy)$ $\text{plotrpoint}(w,dx,dy)$
draw point $(x+dx,y+dy)$ $\text{plotrpoint}(w,dx,dy)$

Postscript Functions

as ploth $\text{psploth}(X = a,b,expr,\{flag\},\{n\})$
as plothraw $\text{psplothraw}(lx,ly,\{flag\})$
as plotdraw $\text{psdraw}([[w_1,x_1,y_1],\dots])$

Based on an earlier version by Joseph H. Silverman
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