Modular forms, modular symbols

(PARI-GP version 2.10.0)

Modular Forms

To be completed later.

Modular Symbols

Let $G = \Gamma_0(N)$, $V_k = \mathbf{Q}[X,Y]_{k-2}$. We let $\Delta = \mathrm{Div}^0(\mathbf{P}^1(\mathbf{Q}))$; an element of Δ is a *path* between cusps of $X_0(N)$ via the identification $[b]-[a] \to$ the path from a to b. A path is coded by the pair $[a,b]$, where a,b are rationals or `oo`, denoting the point at infinity $(1:0)$.

Let $\mathbf{M}_k(G) = \mathrm{Hom}_G(\Delta, V_k) \simeq H^1_c(X_0(G), V_k)$; an element of $\mathbf{M}_k(G)$ is a V_k-valued *modular symbol*. There is a natural decomposition $\mathbf{M}_k(G) = \mathbf{M}_k(G)^+ \oplus \mathbf{M}_k(G)^-$ under the action of the $*$ involution, induced by complex conjugation. The `msinit` function computes either $\mathbf{M}_k$ ($\varepsilon = 0$) or its $\pm$-parts ($\varepsilon = \pm 1$) and fixes a minimal set of $\mathbf{Z}[G]$-generators (g_i) of Δ.

initialize $M = \mathbf{M}_k(\Gamma_0(N))^\varepsilon$	`msinit`$(N,k,\{\varepsilon=0\})$
the level M	`msgetlevel`(M)
the weight k	`msgetweight`(M)
the sign ε	`msgetsign`(M)
$\mathbf{Z}[G]$-generators and relations for Δ	`mspathgens`(M)
Decompose $p=[a,b]$ on the (g_i)	`mspathlog`(M,p)

Create a symbol

Eisenstein symbol attached to cusp c	`msfromcusp`(M,c)
Cuspidal symbol attached to $E/\mathbf{Q}$	`msfromell`(E)
symbol having given Hecke eigenvalues	`msfromhecke`$(M,v,\{H\})$
is s a symbol ?	`msissymbol`(M,s)
the list of all $s(g_i)$	`mseval`(M,s)
evaluate symbol s on path $p=[a,b]$	`mseval`(M,s,p)

Operators

An operator is given by a matrix of a fixed $\mathbf{Q}$-basis. H, if given, is a stable $\mathbf{Q}$-subspace of $\mathbf{M}_k(G)$: operator is restricted to H.

matrix of Hecke operator T_p or U_p	`mshecke`$(M,p,\{H\})$
matrix of Atkin-Lehner w_Q	`msatkinlehner`$(M,Q\{H\})$
matrix of the $*$ involution	`msstar`$(M,\{H\})$

Subspaces

A subspace is given by a structure allowing quick projection and restriction of linear operators. Its fist component is a matrix with integer coefficients whose columns for a $\mathbf{Q}$-basis. If H is a Hecke-stable subspace of $M_k(G)^+$ or $M_k(G)^-$, it can be split into a direct sum of Hecke-simple subspaces. To a simple subspace corresponds a single normalized newform $\sum_n a_n q^n$.

cuspidal subspace $S_k(G)^\varepsilon$	`mscuspidal`(M)
Eisenstein subspace $E_k(G)^\varepsilon$	`mseisenstein`(M)
new part of $S_k(G)^\varepsilon$	`msnew`(M)
split H into simple subspaces (of dim $\le d$)	`mssplit`$(M,H,\{d\})$
$(a_1,\ldots,a_B)$ for attached newform	`msqexpansion`$(M,H,\{B\})$

Overconvergent symbols and p-adic L functions

Let M be a full modular symbol space given by `msinit` and p be a prime. To a classical modular symbol ϕ of level N ($v_p(N) \le 1$), which is an eigenvector for T_p with non-zero eigenvalue a_p, we can attach a p-adic L-function L_p. The function L_p is defined on continuous characters of $\mathrm{Gal}(\mathbf{Q}(\mu_{p^\infty})/\mathbf{Q})$; in GP we allow characters $\langle\chi\rangle^{s_1}\tau^{s_2}$, where (s_1,s_2) are integers, τ is the Teichmüller character and χ is the cyclotomic character.

The symbol ϕ can be lifted to an *overconvergent* symbol Φ, taking values in spaces of p-adic distributions (represented in GP by a list of moments modulo p^n).

`mspadicinit` precomputes data used to lift symbols. If *flag* is given, it speeds up the computation by assuming that $v_p(a_p) = 0$ if *flag* $= 0$ (fastest), and that $v_p(a_p) \ge$ *flag* otherwise (faster as *flag* increases).

`mspadicmoments` computes distributions *mu* attached to Φ allowing to compute L_p to high accuracy.

initialize *Mp* to lift symbols	`mspadicinit`$(M,p,n,\{flag\})$
lift symbol ϕ	`mstooms`(Mp,ϕ)
eval overconvergent symbol Φ on path p	`msomseval`(Mp,Φ,p)
mu for p-adic L-functions	`mspadicmoments`$(Mp,S,\{D=1\})$
$L_p^{(r)}(\chi^s)$, $s=[s_1,s_2]$	`mspadicL`$(mu,\{s=0\},\{r=0\})$
$\hat{L}_p(\tau^i)(x)$	`mspadicseries`$(mu,\{i=0\})$

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