

# Algebraic Number Theory

(PARI-GP version 2.10.0)

## Binary Quadratic Forms

create  $ax^2 + bxy + cy^2$  (distance  $d$ )      `Qfb( $a, b, c, \{d\}$ )`  
reduce  $x$  ( $s = \sqrt{D}$ ,  $l = \lfloor s \rfloor$ )      `qfbred( $x, \{flag\}, \{D\}, \{l\}, \{s\}$ )`  
return  $[y, g]$ ,  $g \in \text{SL}_2(\mathbf{Z})$ ,  $y = g \cdot x$  reduced      `qfbreds12( $x$ )`  
composition of forms       $x*y$  or `qfbnucomp( $x, y, l$ )`  
 $n$ -th power of form       $x^n$  or `qfbnupow( $x, n$ )`  
composition without reduction      `qfbcomprow( $x, y$ )`  
 $n$ -th power without reduction      `qfbpowrow( $x, n$ )`  
prime form of disc.  $x$  above prime  $p$       `qfbprimeform( $x, p$ )`  
class number of disc.  $x$       `qfbclassno( $x$ )`  
Hurwitz class number of disc.  $x$       `qfbhclassno( $x$ )`  
Solve  $Q(x, y) = p$  in integers,  $p$  prime      `qfbsolve( $Q, p$ )`

## Quadratic Fields

quadratic number  $\omega = \sqrt{x}$  or  $(1 + \sqrt{x})/2$       `quadgen( $x$ )`  
minimal polynomial of  $\omega$       `quadpoly( $x$ )`  
discriminant of  $\mathbf{Q}(\sqrt{D})$       `quaddisc( $x$ )`  
regulator of real quadratic field      `quadregulator( $x$ )`  
fundamental unit in real  $\mathbf{Q}(x)$       `quadunit( $x$ )`  
class group of  $\mathbf{Q}(\sqrt{D})$       `quadclassunit( $D, \{flag\}, \{t\}$ )`  
Hilbert class field of  $\mathbf{Q}(\sqrt{D})$       `quadhilbert( $D, \{flag\}$ )`  
... using specific class invariant ( $D < 0$ )      `polclass( $D, \{inv\}$ )`  
ray class field modulo  $f$  of  $\mathbf{Q}(\sqrt{D})$       `quadrays( $D, f, \{flag\}$ )`

## General Number Fields: Initializations

The number field  $K = \mathbf{Q}[X]/(f)$  is given by irreducible  $f \in \mathbf{Q}[X]$ .  
A  $nf$  computes a maximal order and allows operations on elements and ideals. A  $bnf$  adds class group and units. A  $bnr$  is attached to ray class groups and class field theory. A  $rnf$  is attached to relative extensions  $L/K$ .

init number field structure  $nf$       `nfinit( $f, \{flag\}$ )`  
known integer basis  $B$       `nfinit([ $f, B$ ])`  
order maximal at  $vp = [p_1, \dots, p_k]$       `nfinit([ $f, vp$ ])`  
order maximal at all  $p \leq P$       `nfinit([ $f, P$ ])`  
certify maximal order      `nfcertify( $nf$ )`

### nf members:

a monic  $F \in \mathbf{Z}[X]$  defining  $K$        $nf.pol$   
number of real/complex places       $nf.r1/r2/sign$   
discriminant of  $nf$        $nf.disc$   
 $T_2$  matrix       $nf.t2$   
complex roots of  $F$        $nf.roots$   
integral basis of  $\mathbf{Z}_K$  as powers of  $\theta$        $nf.zk$   
different/codifferent       $nf.diff, nf.codiff$   
index  $[\mathbf{Z}_K : \mathbf{Z}[X]/(F)]$        $nf.index$   
recompute  $nf$  using current precision       $nf.newprec(nf)$   
init relative  $rnf$   $L = K[Y]/(g)$        $rnfinit(nf, g)$   
init  $bnf$  structure       $bnfinit(f, \{flag\})$

**bnf members:** same as  $nf$ , plus

underlying  $nf$        $bnf.nf$   
classgroup       $bnf.clgp$   
regulator       $bnf.reg$   
fundamental/torsion units       $bnf.fu, bnf.tu$   
compress a  $bnf$  for storage       $bnf.compress(bnf)$   
recover a  $bnf$  from compressed  $bnfz$        $bnfinit(bnfz)$   
add  $S$ -class group and units, yield  $bnfS$        $bnfcsunit(bnf, S)$   
init class field structure  $bnr$        $bnrinit(bnf, m, \{flag\})$

**bnr members:** same as  $bnf$ , plus

underlying  $bnf$        $bnr.bnf$   
big ideal structure       $bnr.bid$   
modulus       $bnr.mod$   
structure of  $(\mathbf{Z}_K/m)^*$        $bnr.zkst$

## Basic Number Field Arithmetic (nf)

Elements are `t_INT`, `t_FRAC`, `t_POL`, `t_POLMOD`, or `t_COL` (on integral basis  $nf.zk$ ). Basic operations (prefix `nfelt`): (`nfelt`)`add`, `mul`, `pow`, `div`, `diveuc`, `mod`, `divrem`, `val`, `trace`, `norm`  
express  $x$  on integer basis      `nfalgtobasis(nf, x)`  
express element  $x$  as a polmod      `nfbasistoalg(nf, x)`  
complex embeddings of `t_POLMOD`  $x$       `conjvec(x)`  
reverse polmod  $a = A(X) \bmod T(X)$       `modreverse(a)`  
integral basis of field def. by  $f = 0$       `nfbasis(f)`  
field discriminant of field  $f = 0$       `nfdisc(f)`  
smallest poly defining  $f = 0$  (slow)      `polredabs(f, \{flag\})`  
small poly defining  $f = 0$  (fast)      `polredbest(f, \{flag\})`  
random Tschirnhausen transform of  $f$       `poltschirnhaus(f)`  
 $\mathbf{Q}[x]/(f) \subset \mathbf{Q}[x]/(g)$  ? Isomorphic?      `nfisincl(f, g), nfisisom`  
compositum of  $\mathbf{Q}[X]/(f)$ ,  $\mathbf{Q}[X]/(g)$       `polcompositum(f, g, \{flag\})`  
compositum of  $K[X]/(f)$ ,  $K[X]/(g)$       `nfcompositum(nf, f, g, \{flag\})`  
splitting field of  $K$  (degree divides  $d$ )      `nfsplitting(nf, \{d\})`  
subfields (of degree  $d$ ) of  $nf$       `nfsubfields(nf, \{d\})`  
 $d$ -th degree subfield of  $\mathbf{Q}(\zeta_n)$       `polsubcyclo( $n, d, \{v\}$ )`  
roots of unity in  $nf$       `nfrootsof1(nf)`  
roots of  $g$  belonging to  $nf$       `nfroots(\{nf\}, g)`  
factor  $g$  in  $nf$       `nfactor(nf, g)`  
factor  $g$  mod prime  $pr$  in  $nf$       `nfactormod(nf, g, pr)`  
conjugates of a root  $\theta$  of  $nf$       `nfgaloisconj(nf, \{flag\})`  
apply Galois automorphism  $s$  to  $x$       `nfgaloisapply(nf, s, x)`  
quadratic Hilbert symbol (at  $p$ )      `nfhilbert(nf, a, b, \{p\})`

### Linear and algebraic relations

poly of degree  $\leq k$  with root  $x \in \mathbf{C}$       `algdep( $x, k$ )`  
alg. dep. with pol. coeffs for series  $s$       `seralgdep( $s, x, y$ )`  
small linear rel. on coords of vector  $x$       `linddep( $x$ )`

### Dedekind Zeta Function $\zeta_K$ , Hecke $L$ series

$R = [c, w, h]$  in initialization means we restrict  $s \in \mathbf{C}$  to domain  $|\Re(s) - c| < w$ ,  $|\Im(s)| < h$ ;  $R = [w, h]$  encodes  $[1/2, w, h]$  and  $[h]$  encodes  $R = [1/2, 0, h]$  (critical line up to height  $h$ ).

$\zeta_K$  as Dirichlet series,  $N(I) < b$       `dirzetak(nf, b)`  
init  $\zeta_K^{(k)}(s)$  for  $k \leq n$       `L = lfuninit(bnf, R, \{n = 0\})`  
compute  $\zeta_K(s)$  ( $n$ -th derivative)      `lfun(L, s, \{n = 0\})`  
compute  $\Lambda_K(s)$  ( $n$ -th derivative)      `lfunlambda(L, s, \{n = 0\})`

init  $L_K^{(k)}(s, \chi)$  for  $k \leq n$       `L = lfuninit([bnr, chi], R, \{n = 0\})`  
compute  $L_K(s, \chi)$  ( $n$ -th derivative)      `lfun(L, s, \{n\})`  
Artin root number of  $K$       `bnrroottnumber(bnr, chi, \{flag\})`  
 $L(1, \chi)$ , for all  $\chi$  trivial on  $H$       `bnrL1(bnr, \{H\}, \{flag\})`

## Class Groups & Units (bnf, bnr)

Class field theory data  $a_1, \{a_2\}$  is usually  $bnr$  (ray class field),  $bnr, H$  (congruence subgroup) or  $bnr, \chi$  (character on `bnr.clgp`). Any of these define a unique abelian extension of  $K$ .  
remove GRH assumption from  $bnf$       `bnfcertify(bnf)`  
expo. of ideal  $x$  on class gp      `bnfisprincipal(bnf, x, \{flag\})`  
expo. of ideal  $x$  on ray class gp      `bnrisprincipal(bnr, x, \{flag\})`  
expo. of  $x$  on fund. units      `bnfisunit(bnf, x)`  
as above for  $S$ -units      `bnfissunit(bnfs, x)`

signs of real embeddings of  $bnf.fu$       `bnfsignunit(bnf)`  
narrow class group      `bnfnarrow(bnf)`  
**Class Field Theory**  
ray class number for modulus  $m$       `bnrclassno(bnf, m)`  
discriminant of class field      `bnrdisc( $a_1, \{a_2\}$ )`  
ray class numbers,  $l$  list of moduli      `bnrclassolist(bnf, l)`  
discriminants of class fields      `bnrdisclist(bnf, l, \{arch\}, \{flag\})`  
decode output from `bnrdisc`      `bnfdecodemodule(nf, fa)`  
is modulus the conductor?      `bnrisconductor( $a_1, \{a_2\}$ )`  
is class field ( $bnr, H$ ) Galois over  $K^G$       `bnrisgalois(bnr, G, H)`  
action of automorphism on `bnr.gen`      `bnrgaloismatrix(bnr, aut)`  
apply `bnrgaloismatrix`  $M$  to  $H$       `bnrgaloisapply(bnr, M, H)`  
characters on `bnr.clgp` s.t.  $\chi(g_i) = e(v_i)$       `bnrchar(bnr, g, \{v\})`  
conductor of character  $\chi$       `bnrconductor(bnr, chi)`  
conductor of extension      `bnrconductor( $a_1, \{a_2\}, \{flag\}$ )`  
conductor of extension  $K[Y]/(g)$       `rnfconductor(bnf, g)`  
Artin group of extension  $K[Y]/(g)$       `rnfnormgroup(bnr, g)`  
subgroups of  $bnr$ , index  $\leq b$       `subgrouplist(bnr, b, \{flag\})`  
rel. eq. for class field def'd by  $sub$       `rnfkummer(bnr, sub, \{d\})`  
same, using Stark units (real field)      `bnrstark(bnr, sub, \{flag\})`  
is  $a$  an  $n$ -th power in  $K_v$  ?      `nfislocalpower(nf, v, a, n)`  
cyclic  $L/K$  satisf. local conditions      `nfgrunwaldwang(nf, P, D, pl)`

### Logarithmic class group

logarithmic  $\ell$ -class group      `bnflog(bnf, \ell)`  
 $[\tilde{e}(F_v/Q_p), \tilde{f}(F_v/Q_p)]$       `bnflogef(bnf, pr)`  
 $\exp \deg_F(A)$       `bnflogdegree(bnf, A, \ell)`  
is  $\ell$ -extension  $L/K$  locally cyclotomic      `rnfislocalcyclo(rmf)`

**Ideals:** elements, primes, or matrix of generators in HNF

is  $id$  an ideal in  $nf$  ?      `nfisideal(nf, id)`  
is  $x$  principal in  $bnf$  ?      `bnfisprincipal(bnf, x)`  
give  $[a, b]$ , s.t.  $a\mathbf{Z}_K + b\mathbf{Z}_K = x$       `idealtwoelt(nf, x, \{a\})`  
put ideal  $a$  ( $a\mathbf{Z}_K + b\mathbf{Z}_K$ ) in HNF form      `idealhnf(nf, a, \{b\})`  
norm of ideal  $x$       `idealnrm(nf, x)`  
minimum of ideal  $x$  (direction  $v$ )      `idealmin(nf, x, v)`  
LLL-reduce the ideal  $x$  (direction  $v$ )      `idealred(nf, x, \{v\})`

### Ideal Operations

add ideals  $x$  and  $y$       `idealadd(nf, x, y)`  
multiply ideals  $x$  and  $y$       `idealmul(nf, x, y, \{flag\})`  
intersection of ideals  $x$  and  $y$       `idealintersect(nf, x, y, \{flag\})`  
 $n$ -th power of ideal  $x$       `idealpow(nf, x, n, \{flag\})`  
inverse of ideal  $x$       `idealinv(nf, x)`  
divide ideal  $x$  by  $y$       `idealdiv(nf, x, y, \{flag\})`  
Find  $(a, b) \in x \times y$ ,  $a + b = 1$       `idealaddtoone(nf, x, \{y\})`  
coprime integral  $A, B$  such that  $x = A/B$       `idealnumden(nf, x)`

### Primes and Multiplicative Structure

factor ideal  $x$  in  $\mathbf{Z}_K$       `idealfactor(nf, x)`  
expand ideal factorization in  $K$       `idealfactorback(nf, f, \{e\})`  
expand elt factorisation in  $K$       `nfactorback(nf, f, \{e\})`  
decomposition of prime  $p$  in  $\mathbf{Z}_K$       `idealprimedec(nf, p)`  
valuation of  $x$  at prime ideal  $pr$       `idealval(nf, x, pr)`  
weak approximation theorem in  $nf$       `idealchinese(nf, x, y)`  
 $a \in K$ , s.t.  $v_p(a) = v_p(x)$  if  $v_p(x) \neq 0$       `idealappr(nf, x)`  
 $a \in K$  such that  $(a \cdot x, y) = 1$       `idealcoprime(nf, x, y)`  
give  $bid$  = structure of  $(\mathbf{Z}_K/id)^*$       `idealstar(nf, id, \{flag\})`  
structure of  $(1 + \mathfrak{p})/(1 + \mathfrak{p}^k)$       `idealprincipalunits(nf, pr, k)`  
discrete log of  $x$  in  $(\mathbf{Z}_K/bid)^*$       `ideallog(nf, x, bid)`

# Algebraic Number Theory

(PARI-GP version 2.10.0)

**idealstar** of all ideals of norm  $\leq b$       **ideallist**( $nf, b, \{flag\}$ )  
add Archimedean places      **ideallistarch**( $nf, b, \{ar\}, \{flag\}$ )  
init **modpr** structure      **nfmodprinit**( $nf, pr$ )  
project  $t$  to  $\mathbf{Z}_K/pr$       **nfmodpr**( $nf, t, modpr$ )  
lift from  $\mathbf{Z}_K/pr$       **nfmodprlift**( $nf, t, modpr$ )

## Galois theory over $\mathbf{Q}$

Galois group of field  $\mathbf{Q}[x]/(f)$       **polgalois**( $f$ )  
initializes a Galois group structure  $G$       **galoisinit**( $pol, \{den\}$ )  
action of  $p$  in  $\text{nfgaloisconj}$  form      **galoispermopol**( $G, \{p\}$ )  
identify as abstract group      **galoisidentify**( $G$ )  
export a group for GAP/MAGMA      **galoisexport**( $G, \{flag\}$ )  
subgroups of the Galois group  $G$       **galoissubgroups**( $G$ )  
is subgroup  $H$  normal?      **galoisisnormal**( $G, H$ )  
subfields from subgroups      **galoissubfields**( $G, \{flag\}, \{v\}$ )  
fixed field      **galoisfixedfield**( $G, perm, \{flag\}, \{v\}$ )  
Frobenius at maximal ideal  $P$       **idealfrobenius**( $nf, G, P$ )  
ramification groups at  $P$       **idealramgroups**( $nf, G, P$ )  
is  $G$  abelian?      **galoisisabelian**( $G, \{flag\}$ )  
abelian number fields/ $\mathbf{Q}$       **galoissubcyclo**( $N, H, \{flag\}, \{v\}$ )  
query the **galpol** package      **galoisgetpol**( $a, b, \{s\}$ )

## Relative Number Fields (rnf)

Extension  $L/K$  is defined by  $T \in K[x]$ .  
absolute equation of  $L$       **rnfequation**( $nf, T, \{flag\}$ )  
is  $L/K$  abelian?      **rnfisabelian**( $nf, T$ )  
relative **nfalgtobasis**      **rnfalgtobasis**( $rnf, x$ )  
relative **nfbasistoalg**      **rnfbasistoalg**( $rnf, x$ )  
relative **idealhnf**      **rnfidealhnf**( $rnf, x$ )  
relative **idealmul**      **rnfidealmul**( $rnf, x, y$ )  
relative **idealtwoelt**      **rnfidealtwoelt**( $rnf, x$ )

### Lifts and Push-downs

absolute  $\rightarrow$  relative repres. for  $x$       **rnfeltabstorel**( $rnf, x$ )  
relative  $\rightarrow$  absolute repres. for  $x$       **rnfeltreltoabs**( $rnf, x$ )  
lift  $x$  to the relative field      **rnfeltup**( $rnf, x$ )  
push  $x$  down to the base field      **rnfeltdown**( $rnf, x$ )  
idem for  $x$  ideal: (**rnfideal**)**reltoabs**, **abstorel**, **up**, **down**

### Norms and Trace

relative norm of element  $x \in L$       **rnfeltnorm**( $rnf, x$ )  
relative trace of element  $x \in L$       **rnfelttrace**( $rnf, x$ )  
absolute norm of ideal  $x$       **rnfidealnrmabs**( $rnf, x$ )  
relative norm of ideal  $x$       **rnfidealnrmrel**( $rnf, x$ )  
solutions of  $N_{K/\mathbf{Q}}(y) = x \in \mathbf{Z}$       **bnfisintnorm**( $bnf, x$ )  
is  $x \in \mathbf{Q}$  a norm from  $K$ ?      **bnfisnorm**( $bnf, x, \{flag\}$ )  
initialize  $T$  for norm eq. solver      **rnfisnorminit**( $K, pol, \{flag\}$ )  
is  $a \in K$  a norm from  $L$ ?      **rnfisnorm**( $T, a, \{flag\}$ )  
initialize  $t$  for Thue equation solver      **thueinit**( $f$ )  
solve Thue equation  $f(x, y) = a$       **thue**( $t, a, \{sol\}$ )  
characteristic poly. of  $a \bmod T$       **rnfcharpoly**( $nf, T, a, \{v\}$ )

### Factorization

factor ideal  $x$  in  $L$       **rnfidealfactor**( $rnf, x$ )  
 $[S, T]: T_{i,j} \mid S_i; S$  primes of  $K$  above  $p$       **rnfidealprimedec**( $rnf, p$ )

## Maximal order $\mathbf{Z}_L$ as a $\mathbf{Z}_K$ -module

relative **polredbest**      **rnfpolredbest**( $nf, T$ )  
relative Dedekind criterion, prime  $pr$       **rnfdedekind**( $nf, T, pr$ )  
discriminant of relative extension      **rnfdisc**( $nf, T$ )  
pseudo-basis of  $\mathbf{Z}_L$       **rnfpsedobasis**( $nf, T$ )  
**General  $\mathbf{Z}_K$ -modules:**  $M = [\text{matrix}, \text{vec. of ideals}] \subset L$   
relative HNF / SNF      **nfhnf**( $nf, M$ ), **nfsnf**  
multiple of det  $M$       **nfdetint**( $nf, M$ )  
HNF of  $M$  where  $d = nf\text{detint}(M)$       **nfhnfmod**( $x, d$ )  
reduced basis for  $M$       **rnflllgram**( $nf, T, M$ )  
determinant of pseudo-matrix  $M$       **rnfdet**( $nf, M$ )  
Steinitz class of  $M$       **rnfsteinitz**( $nf, M$ )  
 $\mathbf{Z}_K$ -basis of  $M$  if  $\mathbf{Z}_K$ -free, or 0      **rnfhnfbasis**( $bnf, M$ )  
 $n$ -basis of  $M$ , or  $(n+1)$ -generating set      **rnfbasis**( $bnf, M$ )  
is  $M$  a free  $\mathbf{Z}_K$ -module?      **rnfisfree**( $bnf, M$ )

## Associative Algebras

$A$  is a general associative algebra given by a mult. table  $mt$  (over  $\mathbf{Q}$  or  $\mathbf{F}_p$ ); represented by  $al$  from **algtableinit**.  
create  $al$  from  $mt$  (over  $\mathbf{F}_p$ )      **algtableinit**( $mt, \{p = 0\}$ )  
group algebra  $\mathbf{Q}[G]$  (or  $\mathbf{F}_p[G]$ )      **alggroup**( $G, \{p = 0\}$ )

### Properties

is  $(mt, p)$  OK for algtableinit?      **algisassociative**( $mt, \{p = 0\}$ )  
multiplication table  $mt$       **algmultable**( $al$ )  
multiplication table over center      **algrelmultable**( $al$ )  
dimension of  $A$  over prime subfield      **algabsdim**( $al$ )  
characteristic of  $A$       **algchar**( $al$ )  
is  $A$  commutative?      **algiscommutative**( $al$ )  
is  $A$  simple?      **algissimple**( $al$ )  
is  $A$  semi-simple?      **algissemisimple**( $al$ )  
is  $A$  ramified? (at place  $v$ )      **algisramified**( $al, \{v\}$ )  
is  $A$  split? (at place  $v$ )      **algissplit**( $al, \{v\}$ )  
center of  $A$       **algcenter**( $al$ )  
Jacobson radical of  $A$       **algradical**( $al$ )  
radical  $J$  and simple factors of  $A/J$       **algdecomposition**( $al$ )  
simple factors of semi-simple  $A$       **algsimpledec**( $al$ )

### Operations on algebras

create  $A/I$ ,  $I$  two-sided ideal      **algquotient**( $al, I, \{flag = 0\}$ )  
create  $A_1 \otimes A_2$       **algtensor**( $al1, al2$ )  
create subalgebra from basis  $B$       **algsubalg**( $al, B$ )  
...from orthogonal central idempotents  $e$       **algcentralproj**( $al, e$ )  
prime subalgebra of semi-simple  $A$  over  $\mathbf{F}_p$       **algprimesubalg**( $al$ )  
lattice generated by cols. of  $M$       **alglathnf**( $al, M$ )

### Operations on elements

$a + b, a - b, -a$       **algadd**( $al, a, b$ ), **algsub**, **algneg**  
 $a \times b, a \times a$       **algmul**( $al, a, a$ ), **algsqr**  
 $a^n, a^{-1}$       **algpow**( $al, a, n$ ), **alginv**  
is  $x$  invertible? (then set  $z = x^{-1}$ )      **algisinv**( $al, x, \{\&z\}$ )  
find  $z$  such that  $x \times z = y$       **algdivl**( $al, x, y$ )  
find  $z$  such that  $z \times x = y$       **algdivr**( $al, x, y$ )  
does  $z$  s.t.  $x \times z = y$  exist? (set it)      **algisdivl**( $al, x, y, \{\&z\}$ )  
matrix of  $v \mapsto x \cdot v$       **algleftmultable**( $al, x$ )  
absolute norm      **algnorm**( $al, x$ )  
absolute trace      **algtrace**( $al, x$ )  
absolute char. polynomial      **algcharpoly**( $al, x$ )  
given  $a \in A$  and polynomial  $T$ , return  $T(a)$       **algpoleval**( $al, T, a$ )  
random element in a box      **algrandom**( $al, b$ )

## Central Simple Algebras

$A$  is a central simple algebra over a number field  $K$ ; represented by  $al$  from **alginith**;  $K$  is given by a  $nf$  structure.  
create CSA from data      **alginith**( $B, C, \{v\}, \{flag = 0\}$ )  
multiplication table over  $K$        $B = K, C = mt$   
cyclic algebra  $(L/K, \sigma, b)$        $B = rnf, C = [\text{sigma}, b]$   
quaternion algebra  $(a, b)_K$        $B = K, C = [a, b]$   
matrix algebra  $M_d(K)$        $B = K, C = d$   
local Hasse invariants over  $K$        $B = K, C = [d, [PR, HF], HI]$

### Properties

type of  $al$  ( $mt$ , CSA)      **algtype**( $al$ )  
is  $al$  a division algebra? (at place  $v$ )      **algisdivision**( $al, \{v\}$ )  
dimension of  $al$  over its center      **algdim**( $al$ )  
degree of  $A$  ( $= \sqrt{\dim}$ )      **algdegree**( $al$ )  
index of  $A$  over  $K$  (index at  $v$ )      **algindex**( $al, \{v\}$ )  
 $al$  a cyclic algebra  $(L/K, \sigma, b)$ ; return  $\sigma$       **algaut**( $al$ )  
...return  $b$       **algb**( $al$ )  
...return  $L/K$ , as an  $rnf$       **algsplittingfield**( $al$ )  
split  $A$  over an extension of  $K$       **algsplittingdata**( $al$ )  
splitting field of  $A$  as an  $rnf$  over center      **algsplittingfield**( $al$ )  
places of  $K$  at which  $A$  ramifies      **algramifiedplaces**( $al$ )  
Hasse invariants at finite places of  $K$       **alghassef**( $al$ )  
Hasse invariants at infinite places of  $K$       **alghassei**( $al$ )  
Hasse invariant at place  $v$       **alghasse**( $al, v$ )

### Operations on elements

reduced norm      **algnorm**( $al, x$ )  
reduced trace      **algtrace**( $al, x$ )  
reduced char. polynomial      **algcharpoly**( $al, x$ )  
express  $x$  on integral basis      **algalgtobasis**( $al, x$ )  
convert  $x$  to algebraic form      **algbasistoalg**( $al, x$ )  
map  $x \in A$  to  $M_d(L)$ ,  $L$  split. field      **algsplittingmatrix**( $al, x$ )

### Orders

$\mathbf{Z}$ -basis of order  $\mathcal{O}_0$       **algbasis**( $al$ )  
discriminant of order  $\mathcal{O}_0$       **algdisc**( $al$ )  
 $\mathbf{Z}$ -basis of natural order in terms  $\mathcal{O}_0$ 's basis      **alginvbasis**( $al$ )

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