

Pari-GP reference card

(PARI-GP version 2.9.0)

Note: optional arguments are surrounded by braces {}.
To start the calculator, type its name in the terminal: **gp**
To exit **gp**, type **quit**, **\q**, or **<C-D>** at prompt.

Help

describe function	?function
extended description	??keyword
list of relevant help topics	???pattern
name of GP-1.39 function f in GP-2.*	whatnow(f)

Input/Output

previous result, the result before	%, %', %'', etc.
n -th result since startup	%n
separate multiple statements on line	;
extend statement on additional lines	\
extend statements on several lines	{seq1; seq2;}
comment	/* ... */
one-line comment, rest of line ignored	\\ ...

Metacommands & Defaults

set default d to val	default({d}, {val})
toggle timer on/off	#
print time for last result	##
print defaults	\d
set debug level to n	\g n
set memory debug level to n	\gm n
set n significant digits / bits	\p n, \pb n
set n terms in series	\ps n
quit GP	\q
print the list of PARI types	\t
print the list of user-defined functions	\u
read file into GP	\r filename

Debugger / break loop

get out of break loop	break or <C-D>
go up/down n frames	dbg_up({n}), dbg_down
set break point	breakpoint()
examine object o	dbg_x(o)
current error data	dbg_err()
number of objects on heap and their size	getheap()
total size of objects on PARI stack	getstack()

PARI Types & Input Formats

t_INT . Integers; hex, binary	± 31 ; $\pm 0x1F$, $\pm 0b101$
t_REAL . Reals	± 3.14 , $6.022 E23$
t_INTMOD . Integers modulo m	Mod(n, m)
t_FRAC . Rational Numbers	n/m
t_FFELT . Elt in finite field F_q	ffgen(q)
t_COMPLEX . Complex Numbers	$x + y * I$
t_PADIC . p -adic Numbers	$x + O(p^k)$
t_QUAD . Quadratic Numbers	$x + y * \text{quadgen}(D)$
t_POLMOD . Polynomials modulo g	Mod(f, g)
t_POL . Polynomials	$a * x^n + \dots + b$
t_SER . Power Series	$f + O(x^k)$
t_RFRAC . Rational Functions	f/g
t_QFI/t_QFR . Imag/Real binary quad. form	Qfb($a, b, c, \{d\}$)
t_VEC/t_COL . Row/Column Vectors	$[x, y, z]$, $[x, y, z]~$
t_VEC integer range	$[1..10]$

t_VECSMALL . Vector of small ints	Vecsmall($[x, y, z]$)
t_MAT . Matrices	$[a, b; c, d]$
t_LIST . Lists	List($[x, y, z]$)
t_STR . Strings	"abc"
t_INFINITY . $\pm\infty$	+oo, -oo

Reserved Variable Names

$\pi = 3.14\dots$, $\gamma = 0.57\dots$, $C = 0.91\dots$	Pi, Euler, Catalan
square root of -1	I
Landau's big-oh notation	O

Information about an Object

PARI type of object x	type(x)
length of x / size of x in memory	#x, sizebyte(x)
real precision / bit precision of x	precision(x), bitprecision
p -adic, series prec. of x	padicprec(x), serprec

Operators

basic operations	+, -, *, /, ^, sqr
$i=i+1$, $i=i-1$, $i=i*j$, ...	i++, i--, i*=j, ...
euclidean quotient, remainder	$x \backslash y$, $x \backslash y$, $x \% y$, divrem(x, y)
shift x left or right n bits	$x \ll n$, $x \gg n$ or shift($x, \pm n$)
multiply by 2^n	shiftmul(x, n)
comparison operators	<=, <, >=, >, ==, !=, ==, lex, cmp
boolean operators (or, and, not)	, &&, !
bit operations	bitand, bitneg, bitor, bitxor, bitnegimply
sign of $x = -1, 0, 1$	sign(x)
maximum/minimum of x and y	max, min(x, y)
derivative of f	f'
differential operator	diffop($f, v, d, \{n = 1\}$)
quote operator (formal variable)	'x
assignment	x = value
simultaneous assignment $x \leftarrow v_1, y \leftarrow v_2$	[x, y] = v

Select Components

n -th component of x	component(x, n)
n -th component of vector/list x	$x[n]$
components $a, a+1, \dots, b$ of vector x	$x[a..b]$
(m, n) -th component of matrix x	$x[m, n]$
row m or column n of matrix x	$x[m,]$, $x[, n]$
numerator/denominator of x	numerator(x), denominator

Random Numbers

random integer/prime in $[0, N[$	random(N), randomprime
get/set random seed	getrand, setrand(s)

Conversions

to vector, matrix, vec. of small ints	Col/Vec, Mat, Vecsmall
to list, set, map, string	List, Set, Map, Str
create PARI object ($x \bmod y$)	Mod(x, y)
make x a polynomial of v	Pol($x, \{v\}$)
as Pol, etc., starting with constant term	Polrev, Vecrev, Colrev
make x a power series of v	Ser($x, \{v\}$)
string from bytes / from format+args	Strchr, Strprintf
TeX string	Strtex(x)
convert x to simplest possible type	simplify(x)
object x with real precision n	precision(x, n)
object x with bit precision n	bitprecision(x, n)
set precision to p digits in dynamic scope	localprec(p)
set precision to p bits in dynamic scope	localbitprec(p)

Conjugates and Lifts

conjugate of a number x	conj(x)
norm of x , product with conjugate	norm(x)
L^p norm of x (L^∞ if no p)	normlp($x, \{p\}$)
square of L^2 norm of x	norml2(x)
lift of x from Mods and p -adics	lift, centerlift(x)
recursive lift	liftall
lift all t_INT and t_PADIC (\rightarrow t_INT)	liftint
lift all t_POLMOD (\rightarrow t_POL)	lifttpol

Lists, Sets & Maps

Sets (= row vector with strictly increasing entries w.r.t. cmp)	
intersection of sets x and y	setintersect(x, y)
set of elements in x not belonging to y	setminus(x, y)
union of sets x and y	setunion(x, y)
does y belong to the set x	setsearch($x, y, \{flag\}$)
set of all $f(x, y)$, $x \in X, y \in Y$	setbinop(f, X, Y)
is x a set ?	setisset(x)

Lists . create empty list: $L = \text{List}()$	
append x to list L	listput($L, x, \{i\}$)
remove i -th component from list L	listpop($L, \{i\}$)
insert x in list L at position i	listinsert(L, x, i)
sort the list L in place	listsort($L, \{flag\}$)

Maps . create empty dictionary: $M = \text{Map}()$	
attach value v to key k	mapput(M, k, v)
recover value attach to key k or error	mapget(M, k)
is key k in the dict ? (set v to $M(k)$)	mapisdefined($M, k, \{\&v\}$)
remove k from map domain	mapdelete(M, k)

GP Programming

User functions and closures
 x, y are formal parameters; y defaults to Pi if parameter omitted;
 z, t are local variables (lexical scope), z initialized to 1.

```
fun(x, y=Pi) = my(z=1, t); seq
fun = (x, y=Pi) -> my(z=1, t); seq
attach a help message to f      addhelp(f)
undefine symbol s (also kills help)  kill(s)
Control Statements ( $X$ : formal parameter in expression  $seq$ )
if  $a \neq 0$ , evaluate  $seq_1$ , else  $seq_2$   if(a, {seq1}, {seq2})
eval.  $seq$  for  $a \leq X \leq b$ 
...for primes  $a \leq X \leq b$           forprime(X = a, b, seq)
...for composites  $a \leq X \leq b$     forcomposite(X = a, b, seq)
...for  $a \leq X \leq b$  stepping  $s$     forstep(X = a, b, s, seq)
...for  $X$  dividing  $n$                 fordiv(n, X, seq)
multivariable for, lex ordering     forvec(X = v, seq)
loop over partitions of  $n$           forpart(p = n, seq)
loop over vectors  $v, q(v) \leq B; q > 0$   forqfvec(v, q, b, seq)
loop over  $H < G$  finite abelian group  forsubgroup(H = G)
```

```
evaluate seq until  $a \neq 0$ 
while  $a \neq 0$ , evaluate seq          while(a, seq)
exit  $n$  innermost enclosing loops   break({n})
start new iteration of  $n$ -th enclosing loop  next({n})
return  $x$  from current subroutine       return({x})
```

Exceptions, warnings	
raise an exception / warn	error(), warning()
type of error message E	errname(E)
try seq_1 , evaluate seq_2 on error	iferr(seq_1, E, seq_2)

Functions with closure arguments / results

select from v according to f	<code>select(f, v)</code>
apply f to all entries in v	<code>apply(f, v)</code>
evaluate $f(a_1, \dots, a_n)$	<code>call(f, a)</code>
evaluate $f(\dots f(f(a_1, a_2), a_3) \dots, a_n)$	<code>fold(f, a)</code>
calling function as closure	<code>self()</code>

Sums & Products

sum $X = a$ to $X = b$, initialized at x	<code>sum(X = a, b, expr, {x})</code>
sum entries of vector v	<code>vecsum(v)</code>
sum $expr$ over divisors of n	<code>sumdiv(n, X, expr)</code>
... assuming $expr$ multiplicative	<code>sumdivmult(n, X, expr)</code>
product $a \leq X \leq b$, initialized at x	<code>prod(X = a, b, expr, {x})</code>
product over primes $a \leq X \leq b$	<code>prodeuler(X = a, b, expr)</code>

Sorting

sort x by k -th component	<code>vecsort(x, {k}, {fl = 0})</code>
min. m of x ($m = x[i]$), max.	<code>vecmin(x, {&i}), vecmax</code>
does y belong to x , sorted wrt. f	<code>vecsearch(x, y, {f})</code>

Input/Output

print with/without $\backslash n$, $\text{T}_\text{E}_\text{X}$ format	<code>print, print1, printtex</code>
print fields with separator	<code>printsep(sep, ...), printsep1</code>
formatted printing	<code>printf()</code>
write $args$ to file	<code>write, write1, writetex(file, args)</code>
write x in binary format	<code>writebin(file, x)</code>
read file into GP	<code>read({file})</code>
... return as vector of lines	<code>readvec({file})</code>
... return as vector of strings	<code>readstr({file})</code>
read a string from keyboard	<code>input()</code>

Timers

CPU time in ms and reset timer	<code>gettime()</code>
CPU time in ms since gp startup	<code>getabstime()</code>
time in ms since UNIX Epoch	<code>getwalltime()</code>
timeout command after s seconds	<code>alarm(s, expr)</code>

Interface with system

allocates a new stack of s bytes	<code>allocatemem({s})</code>
alias old to new	<code>alias(new, old)</code>
install function from library	<code>install(f, code, {gpf}, {lib})</code>
execute system command a	<code>system(a)</code>
as above, feed result to GP	<code>extern(a)</code>
as above, return GP string	<code>externstr(a)</code>
get $\$VAR$ from environment	<code>getenv("VAR")</code>
expand env. variable in string	<code>Strexpand(x)</code>

Parallel evaluation

These functions evaluate their arguments in parallel (pthreads or MPI); args. must not access global variables and must be free of side effects. Enabled if threading engine is not *single* in gp header.

evaluate f on $x[1], \dots, x[n]$	<code>parapply(f, x)</code>
evaluate closures $f[1], \dots, f[n]$	<code>pareval(f)</code>
as select	<code>parselect(f, A, {flag})</code>
as sum	<code>parsum(i = a, b, expr, {x})</code>
as vector	<code>parvector(n, i, {expr})</code>
eval f for $i = a, \dots, b$	<code>parfor(i = a, {b}, f, {r}, {f2})</code>
... for p prime in $[a, b]$	<code>parforprime(p = a, {b}, f, {r}, {f2})</code>
... multivariate	<code>parforvec(X = v, f, {r}, {f2}, {flag})</code>
declare x as inline (allows to use as global)	<code>inline(x)</code>
stop inlining	<code>uninline()</code>

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Linear Algebra

dimensions of matrix x	<code>matsize(x)</code>
concatenation of x and y	<code>concat(x, {y})</code>
extract components of x	<code>vecextract(x, y, {z})</code>
transpose of vector or matrix x	<code>mattranspose(x)</code> or $x-$
adjoint of the matrix x	<code>matadjoint(x)</code>
eigenvectors/values of matrix x	<code>mateigen(x)</code>
characteristic/minimal polynomial of x	<code>charpoly(x), minpoly</code>
trace/determinant of matrix x	<code>trace(x), matdet</code>
Frobenius form of x	<code>matfrobenius(x)</code>
QR decomposition	<code>matqr(x)</code>
apply <code>matqr</code> 's transform to v	<code>mathousholder(Q, v)</code>

Constructors & Special Matrices

$\{g(x) : x \in v \text{ s.t. } f(x)\}$	<code>[g(x) x <- v, f(x)]</code>
$\{x : x \in v \text{ s.t. } f(x)\}$	<code>[x x <- v, f(x)]</code>
$\{g(x) : x \in v\}$	<code>[g(x) x <- v]</code>
row vec. of $expr$ eval'ed at $1 \leq i \leq n$	<code>vector(n, {i}, {expr})</code>
col. vec. of $expr$ eval'ed at $1 \leq i \leq n$	<code>vectorv(n, {i}, {expr})</code>
vector of small ints	<code>vectorsmall(n, {i}, {expr})</code>
$[c, c \cdot x, \dots, c \cdot x^n]$	<code>powers(x, n, {c = 1})</code>
matrix $1 \leq i \leq m, 1 \leq j \leq n$	<code>matrix(m, n, {i}, {j}, {expr})</code>
define matrix by blocks	<code>matconcat(B)</code>
diagonal matrix with diagonal x	<code>matdiagonal(x)</code>
is x diagonal?	<code>matisdiagonal(x)</code>
$x \cdot \text{matdiagonal}(d)$	<code>matmuldiagonal(x, d)</code>
$n \times n$ identity matrix	<code>matid(n)</code>
Hessenberg form of square matrix x	<code>mathess(x)</code>
$n \times n$ Hilbert matrix $H_{ij} = (i + j - 1)^{-1}$	<code>mathilbert(n)</code>
$n \times n$ Pascal triangle	<code>matpascal(n - 1)</code>
companion matrix to polynomial x	<code>matcompanion(x)</code>
Sylvester matrix of x	<code>pol sylvestermatrix(x)</code>

Gaussian elimination

kernel of matrix x	<code>matker(x, {flag})</code>
intersection of column spaces of x and y	<code>matintersect(x, y)</code>
solve $M * X = B$ (M invertible)	<code>matsolve(M, B)</code>
as solve, modulo D (col. vector)	<code>matsolvemod(M, D, B)</code>
one sol of $M * X = B$	<code>matinverseimage(M, B)</code>
basis for image of matrix x	<code>matimage(x)</code>
columns of x not in <code>matimage</code>	<code>matimagecompl(x)</code>
supplement columns of x to get basis	<code>mat supplement(x)</code>
rows, cols to extract invertible matrix	<code>matindexrank(x)</code>
rank of the matrix x	<code>matrank(x)</code>

Lattices & Quadratic Forms

Quadratic forms

evaluate ${}^t x Q y$	<code>qfeval({Q = id}, x, y)</code>
evaluate ${}^t x Q x$	<code>qfeval({Q = id}, x)</code>
signature of quad form ${}^t y * x * y$	<code>qf sign(x)</code>
decomp into squares of ${}^t y * x * y$	<code>qfgaussred(x)</code>
eigenvalues/vectors for real symmetric x	<code>qfjacob(x)</code>

HNF and SNF

upper triangular Hermite Normal Form	<code>mathnf(x)</code>
HNF of x where d is a multiple of $\det(x)$	<code>mathnfmod(x, d)</code>
multiple of $\det(x)$	<code>matdetint(x)</code>
HNF of $(x \mid \text{diagonal}(D))$	<code>mathnfmod(x, D)</code>
elementary divisors of x	<code>mathsnf(x)</code>
elementary divisors of $\mathbf{Z}[a]/(f'(a))$	<code>poldiscreduced(f)</code>
integer kernel of x	<code>matkerint(x)</code>
\mathbf{Z} -module \leftrightarrow \mathbf{Q} -vector space	<code>matrixqz(x, p)</code>

Lattices

LLL-algorithm applied to columns of x	<code>qflll(x, {flag})</code>
... for Gram matrix of lattice	<code>qflllgram(x, {flag})</code>
find up to m sols of $q\text{norm}(x, y) \leq b$	<code>qfminim(x, b, m)</code>
$v, v[i] :=$ number of y s.t. $q\text{norm}(x, y) = i$	<code>qfrep(x, B, {flag})</code>
perfection rank of x	<code>qfperfection(x)</code>
find isomorphism between q and Q	<code>qfisom(q, Q)</code>
precompute for isomorphism test with q	<code>qfisominit(q)</code>
automorphism group of q	<code>qfauto(q)</code>
convert <code>qfauto</code> for GAP/Magma	<code>qfautoexport(G, {flag})</code>
orbits of V under $G \subset \text{GL}(V)$	<code>qforbits(G, V)</code>

Polynomials & Rational Functions

all defined polynomial variables	<code>variables()</code>
get var. of highest priority (higher than v)	<code>varhigher(name, {v})</code>
... of lowest priority (lower than v)	<code>varlower(name, {v})</code>

Coefficients, variables and basic operators

degree of f	<code>poldegree(f)</code>
coeff. of degree n of f , leading coeff.	<code>polcoeff(f, n), pollead</code>
main variable / all variables in f	<code>variable(f), variables(f)</code>
replace x by y in f	<code>subst(f, x, y)</code>
evaluate f replacing vars by their value	<code>eval(f)</code>
replace polynomial expr. $T(x)$ by y in f	<code>substpol(f, T, y)</code>
replace x_1, \dots, x_n by y_1, \dots, y_n in f	<code>substvec(f, x, y)</code>
reciprocal polynomial $x^{\deg f} f(1/x)$	<code>polrecip(f)</code>
gcd of coefficients of f	<code>content(f)</code>
derivative of f w.r.t. x	<code>deriv(f, {x})</code>
formal integral of f w.r.t. x	<code>intformal(f, {x})</code>
formal sum of f w.r.t. x	<code>sumformal(f, {x})</code>

Constructors & Special Polynomials

interpolating pol. eval. at a	<code>polinterpolate(X, {Y}, {a})</code>
$P_n, T_n/U_n, H_n$	<code>pollegendre, polchebyshev, polhermite</code>
n -th cyclotomic polynomial Φ_n	<code>polcyclo(n, {v})</code>
return n if $f = \Phi_n$, else 0	<code>poliscyclo(f)</code>
is f a product of cyclotomic polynomials?	<code>poliscycloprod(f)</code>
Zagier's polynomial of index (n, m)	<code>polzagier(n, m)</code>

Resultant, elimination

discriminant of polynomial f	<code>poldisc(f)</code>
resultant $R = \text{Res}_v(f, g)$	<code>polresultant(f, g, {v})</code>
$[u, v, R], xu + yv = \text{Res}_v(f, g)$	<code>polresultantext(x, y, {v})</code>
solve Thue equation $f(x, y) = a$	<code>thue(t, a, {sol})</code>
initialize t for Thue equation solver	<code>thueinit(f)</code>

Based on an earlier version by Joseph H. Silverman

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Send comments and corrections to (Karim.Belabas@math.u-bordeaux.fr)

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Roots and Factorization

complex roots of f `polroots(f)`
 number of real roots of f (in $[a, b]$) `polsturm(f, {[a, b]})`
 real roots of f (in $[a, b]$) `polrootsreal(f, {[a, b]})`
 symmetric powers of roots of f up to n `polSYM(f, n)`
 Graeffe transform of f , $g(x^2) = f(x)f(-x)$ `polgraeffe(f)`
 factor f `factor(f)`
 factor f mod p / roots `factormod(f, p), polrootsmod`
 ... using Cantor-Zassenhaus `factorcantor(f, p)`
 factor f over \mathbf{F}_{p^a} / roots `factorff(f, p, a), polrootsff`
 factor f over \mathbf{Q}_p / roots `factorpadic(f, p, r), polrootspadic`
 cyclotomic factors of $f \in \mathbf{Q}[X]$ `polcyclofactors(f)`
 find irreducible $T \in \mathbf{F}_p[x]$, $\deg T = n$ `ffinit(p, n, {x})`
 # {monic irred. $T \in \mathbf{F}_q[x]$, $\deg T = n$ } `ffnbirred(q, n)`
 p -adic root of f congruent to a mod p `padicappr(f, a)`
 Newton polygon of f for prime p `newtonpoly(f, p)`
 Hensel lift $A/\text{lc}(A) = \prod_i B[i] \pmod{p^e}$ `polhensellift(A, B, p, e)`
 extensions of \mathbf{Q}_p of degree N `padicfields(p, N)`

Formal & p -adic Series

truncate power series or p -adic number valuation of x at p `truncate(x), valuation(x, p)`
Dirichlet and Power Series
 Taylor expansion around 0 of f w.r.t. x `taylor(f, x)`
 $\sum a_k b_k t^k$ from $\sum a_k t^k$ and $\sum b_k t^k$ `serconvol(a, b)`
 $f = \sum a_k t^k$ from $\sum (a_k/k!) t^k$ `serlaplace(f)`
 reverse power series F so $F(f(x)) = x$ `serreverse(f)`
 Dirichlet series multiplication / division `dirmul, dirdiv(x, y)`
 Dirichlet Euler product (b terms) `direuler(p = a, b, expr)`

Transcendental and p -adic Functions

real, imaginary part of x `real(x), imag(x)`
 absolute value, argument of x `abs(x), arg(x)`
 square/ n th root of x `sqrt(x), sqrtn(x, n, {&z})`
 trig functions `sin, cos, tan, cotan, sinc`
 inverse trig functions `asin, acos, atan`
 hyperbolic functions `sinh, cosh, tanh, cotanh`
 inverse hyperbolic functions `asinh, acosh, atanh`
 $\log(x)$, e^x , $e^x - 1$ `log, exp, expm1`
 Euler Γ function, $\log \Gamma$, Γ'/Γ `gamma, lngamma, psi`
 half-integer gamma function $\Gamma(n + 1/2)$ `gammah(n)`
 Riemann's zeta $\zeta(s) = \sum n^{-s}$ `zeta(s)`
 multiple zeta value (MZV), $\zeta(s_1, \dots, s_k)$ `zetamult(s)`
 incomplete Γ function ($y = \Gamma(s)$) `incgam(s, x, {y})`
 complementary incomplete Γ `incgamc(s, x)`
 exponential integral $\int_x^\infty e^{-t}/t dt$ `eint1(x)`
 error function $2/\sqrt{\pi} \int_x^\infty e^{-t^2} dt$ `erfc(x)`
 dilogarithm of x `dilog(x)`
 m -th polylogarithm of x `polylog(m, x, {flag})`
 U -confluent hypergeometric function `hyperu(a, b, u)`
 Bessel $J_n(x)$, $J_{n+1/2}(x)$ `besselj(n, x), besseljh(n, x)`
 Bessel I_ν , K_ν , H_ν^1 , H_ν^2 , N_ν `(bessel)i, k, h1, h2, n`
 Lambert W : x s.t. $xe^x = y$ `lambertw(y)`
 Teichmuller character of p -adic x `teichmuller(x)`

Iterations, Sums & Products

Numerical integration for meromorphic functions

Behaviour at endpoint for Double Exponential methods: either a scalar ($a \in \mathbf{C}$, regular) or $\pm\infty$ (decreasing at least as x^{-2}) or $(x - a)^{-\alpha}$ singularity `[a, alpha]`
 exponential decrease $e^{-\alpha|x|}$ `[±∞, alpha], alpha > 0`
 slow decrease $|x|^\alpha$ `... alpha < -1`
 oscillating as $\cos(kx)$ `alpha = kI, k > 0`
 oscillating as $\sin(kx)$ `alpha = -kI, k > 0`
 numerical integration `intnum(x = a, b, f, {T})`
 weights T for intnum `intnuminit(a, b, {m})`
 weights T incl. kernel K `intfuncinit(a, b, K, {m})`
 integrate $(2i\pi)^{-1}f$ on circle $|z - a| = R$ `intcirc(x = a, R, f, {T})`

Other integration methods

n -point Gauss-Legendre `intnumgauss(x = a, b, f, {n})`
 weights for n -point Gauss-Legendre `intnumgaussinit({n})`
 Romberg integration (low accuracy) `intnumromb(x = a, b, f, {flag})`

Numerical summation

sum of series $f(n)$, $n \geq a$ (low accuracy) `suminf(n = a, expr)`
 sum of alternating/positive series `sumalt, sumpos`
 sum of series using Euler-Maclaurin `sumnum(n = a, f, {T})`
 weights for sumnum, a as in DE `sumnuminit({∞, a})`
 sum of series by Monien summation `sumnummonien(n = a, f, {T})`
 weights for sumnummonien `sumnummonieninit({∞, a})`

Products

product $a \leq X \leq b$, initialized at x `prod(X = a, b, expr, {x})`
 product over primes $a \leq X \leq b$ `prodeuler(X = a, b, expr)`
 infinite product $a \leq X \leq \infty$ `prodinf(X = a, expr)`

Other numerical methods

real root of f in $[a, b]$; bracketed root `solve(X = a, b, f)`
 ... by interval splitting `solvestep(X = a, b, f, {flag = 0})`
 limit of $f(t)$, $t \rightarrow \infty$ `limitnum(f, {k}, {alpha})`
 asymptotic expansion of f at ∞ `asymnum(f, {k}, {alpha})`
 numerical derivation w.r.t. x : $f'(a)$ `derivnum(x = a, f)`
 evaluate continued fraction F at t `contfracval(F, t, {L})`
 power series to cont. fraction (L terms) `contfracinit(S, {L})`
 Padé approximant (deg. denom. $\leq B$) `bestapprPade(S, {B})`

Elementary Arithmetic Functions

vector of binary digits of $|x|$ `binary(x)`
 bit number n of integer x `bittest(x, n)`
 Hamming weight of integer x `hammingweight(x)`
 digits of integer x in base B `digits(x, {B = 10})`
 sum of digits of integer x in base B `sumdigits(x, {B = 10})`
 integer from digits `fromdigits(v, {B = 10})`
 ceiling/floor/fractional part `ceil, floor, frac`
 round x to nearest integer `round(x, {&e})`
 truncate x `truncate(x, {&e})`
 gcd/LCM of x and y `gcd(x, y), lcm(x, y)`
 gcd of entries of a vector/matrix `content(x)`

Primes and Factorization

extra prime table `addprimes()`
 add primes in v to prime table `addprimes(v)`
 remove primes from prime table `removeprimes(v)`
 Chebyshev $\pi(x)$, n -th prime p_n `primepi(x), prime(n)`
 vector of first n primes `primes(n)`
 smallest prime $\geq x$ `nextprime(x)`
 largest prime $\leq x$ `precprime(x)`
 factorization of x `factor(x, {lim})`
 ... selecting specific algorithms `factorint(x, {flag = 0})`
 $n = df^2$, d squarefree/fundamental `core(n, {fl}), coredisc`
 recover x from its factorization `factorback(f, {e})`
 $x \in \mathbf{Z}$, $|x| \leq X$, $\gcd(N, P(x)) \geq N$ `zncoppersmith(P, N, X, {B})`

Divisors and multiplicative functions

number of prime divisors $\omega(n)$ / $\Omega(n)$ `omega(n), bigomega`
 divisors of n / number of divisors $\tau(n)$ `divisors(n), numdiv`
 sum of (k -th powers of) divisors of n `sigma(n, {k})`
 Möbius μ -function `moebius(x)`
 Ramanujan's τ -function `ramanujantau(x)`

Combinatorics

factorial of x `x!` or `factorial(x)`
 binomial coefficient $\binom{x}{y}$ `binomial(x, y)`
 Bernoulli number B_n as real/rational `bernreal(n), bernfrac`
 Bernoulli polynomial $B_n(x)$ `bernpol(n, {x})`
 n -th Fibonacci number `fibonacci(n)`
 Stirling numbers $s(n, k)$ and $S(n, k)$ `stirling(n, k, {flag})`
 number of partitions of n `numpart(n)`
 k -th permutation on n letters `numtoperm(n, k)`
 convert permutation to (n, k) form `permtotnum(v)`

Multiplicative groups $(\mathbf{Z}/N\mathbf{Z})^*$, \mathbf{F}_q^*

Euler ϕ -function `eulerphi(x)`
 multiplicative order of x (divides ϕ) `znorder(x, {o}), fforder`
 primitive root mod q / $x \pmod q$ `znprimroot(q), fprimroot(x)`
 structure of $(\mathbf{Z}/n\mathbf{Z})^*$ `znstar(n)`
 discrete logarithm of x in base g `znlog(x, g, {o}), fflag`
 Kronecker-Legendre symbol $\left(\frac{x}{y}\right)$ `kronecker(x, y)`
 quadratic Hilbert symbol (at p) `hilbert(x, y, {p})`

Miscellaneous

integer square / n -th root of x `sqrtint(x), sqrtnint(x, n)`
 largest integer e s.t. $b^e \leq b$, $e = \lfloor \log_b(x) \rfloor$ `logint(x, b, {&z})`
 CRT: solve $z \equiv x$ and $z \equiv y$ `chinese(x, y)`
 minimal u, v so $ux + yv = \gcd(x, y)$ `gcdext(x, y)`
 continued fraction of x `contfrac(x, {b}, {lmax})`
 last convergent of continued fraction x `contfracpnqn(x)`
 rational approximation to x (den. $\leq B$) `bestappr(x, {B}k)`

Characters

Let $cyc = [d_1, \dots, d_k]$ represent an abelian group $G = \oplus (\mathbf{Z}/d_j\mathbf{Z}) \cdot g_j$ or any structure G affording a `.cyc` method; e.g. `idealstar(, q)` for Dirichlet characters. A character χ is coded by $[c_1, \dots, c_k]$ such that $\chi(g_j) = e(n_j/d_j)$.
 $\chi \cdot \psi$; χ^{-1} ; $\chi \cdot \psi^{-1}$ `charmul, charconj, chardiv`
 order of χ `charorder(cyc, chi)`
 kernel of χ `charker(cyc, chi)`
 $\chi(x)$, G a GP group structure `chareval(G, chi, x, {z})`

Dirichlet Characters

initialize $G = (\mathbf{Z}/q\mathbf{Z})^*$ $G = \text{idealstar}(q)$
 is χ odd? $\text{zncharisodd}(G, \chi)$
 real $\chi \rightarrow$ Kronecker symbol (D/\cdot) $\text{znchartokronecker}(G, \chi)$
 induce $\chi \in \hat{G}$ to $\mathbf{Z}/N\mathbf{Z}$ $\text{zncharinduce}(G, \chi, N)$

Conrey labelling

Conrey label $m \in (\mathbf{Z}/q\mathbf{Z})^* \rightarrow$ character $\text{znconreychar}(G, m)$
 character \rightarrow Conrey label $\text{znconreyexp}(G, \chi)$
 log on Conrey generators $\text{znconreylog}(G, m)$
 conductor of χ (χ_0 primitive) $\text{znconreyconductor}(G, \chi, \{\chi_0\})$

True-False Tests

is x the disc. of a quadratic field? $\text{isfundamental}(x)$
 is x a prime? $\text{isprime}(x)$
 is x a strong pseudo-prime? $\text{ispseudoprime}(x)$
 is x square-free? $\text{issquarefree}(x)$
 is x a square? $\text{issquare}(x, \{\&n\})$
 is x a perfect power? $\text{ispower}(x, \{k\}, \{\&n\})$
 is x a perfect power of a prime? ($x = p^n$) $\text{isprimepower}(x, \&n)$
 ... of a pseudoprime? $\text{ispseudoprimepower}(x, \&n)$
 is x powerful? $\text{ispowerful}(x)$
 is x a totient? ($x = \varphi(n)$) $\text{istotient}(x, \{\&n\})$
 is x a polygonal number? ($x = P(s, n)$) $\text{ispolygonal}(x, s, \{\&n\})$
 is pol irreducible? $\text{polisirreducible}(pol)$

Graphic Functions

crude graph of $expr$ between a and b $\text{plot}(X = a, b, expr)$
High-resolution plot (immediate plot)
 plot $expr$ between a and b $\text{ploth}(X = a, b, expr, \{flag\}, \{n\})$
 plot points given by lists lx, ly $\text{plothraw}(lx, ly, \{flag\})$
 terminal dimensions $\text{plothsizes}()$

Rectwindow functions

init window w , with size x, y $\text{plotinit}(w, x, y)$
 erase window w $\text{plotkill}(w)$
 copy w to w_2 with offset (dx, dy) $\text{plotcopy}(w, w_2, dx, dy)$
 clips contents of w $\text{plotclip}(w)$
 scale coordinates in w $\text{plotscale}(w, x_1, x_2, y_1, y_2)$
 ploth in w $\text{plotrecth}(w, X = a, b, expr, \{flag\}, \{n\})$
 plothraw in w $\text{plotrecthraw}(w, data, \{flag\})$
 draw window w_1 at $(x_1, y_1), \dots$ $\text{plotdraw}([[w_1, x_1, y_1], \dots])$

Low-level Rectwindow Functions

set current drawing color in w to c $\text{plotcolor}(w, c)$
 current position of cursor in w $\text{plotcursor}(w)$
 write s at cursor's position $\text{plotstring}(w, s)$
 move cursor to (x, y) $\text{plotmove}(w, x, y)$
 move cursor to $(x + dx, y + dy)$ $\text{plotrmove}(w, dx, dy)$
 draw a box to (x_2, y_2) $\text{plotbox}(w, x_2, y_2)$
 draw a box to $(x + dx, y + dy)$ $\text{plotrbox}(w, dx, dy)$
 draw polygon $\text{plotlines}(w, lx, ly, \{flag\})$
 draw points $\text{plotpoints}(w, lx, ly)$
 draw line to $(x + dx, y + dy)$ $\text{plotrline}(w, dx, dy)$
 draw point $(x + dx, y + dy)$ $\text{plotrpoint}(w, dx, dy)$
 draw point $(x + dx, y + dy)$ $\text{plotrpoint}(w, dx, dy)$

Postscript Functions

as ploth $\text{psploth}(X = a, b, expr, \{flag\}, \{n\})$
 as plothraw $\text{psplothraw}(lx, ly, \{flag\})$
 as plotdraw $\text{psdraw}([[w_1, x_1, y_1], \dots])$

Based on an earlier version by Joseph H. Silverman

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Send comments and corrections to Karim.Belabas@math.u-bordeaux.fr