

Pari-GP reference card

(PARI-GP version 2.9.0)

Note: optional arguments are surrounded by braces {}.

To start the calculator, type its name in the terminal: **gp**

To exit **gp**, type **quit**, **\q**, or **<C-D>** at prompt.

Help

describe function	? <i>function</i>
extended description	?? <i>keyword</i>
list of relevant help topics	??? <i>pattern</i>
name of GP-1.39 function <i>f</i> in GP-2.*	whatnow(<i>f</i>)

Input/Output

previous result, the result before	%, %', %'', etc.
<i>n</i> -th result since startup	% <i>n</i>
separate multiple statements on line	;
extend statement on additional lines	\
extend statements on several lines	{ <i>seq</i> ₁ ; <i>seq</i> ₂ ;
comment	/* ... */
one-line comment, rest of line ignored	\\ ...

Metacommands & Defaults

set default <i>d</i> to <i>val</i>	default({ <i>d</i> },{ <i>val</i> })
toggle timer on/off	#
print time for last result	##
print defaults	\d
set debug level to <i>n</i>	\g <i>n</i>
set memory debug level to <i>n</i>	\gm <i>n</i>
set <i>n</i> significant digits / bits	\p <i>n</i> , \pb <i>n</i>
set <i>n</i> terms in series	\ps <i>n</i>
quit GP	\q
print the list of PARI types	\t
print the list of user-defined functions	\u
read file into GP	\r <i>filename</i>

Debugger / break loop

get out of break loop	break or <C-D>
go up/down <i>n</i> frames	dbg_up({ <i>n</i> }), dbg_down
set break point	breakpoint()
examine object <i>o</i>	dbg_x(<i>o</i>)
current error data	dbg_err()
number of objects on heap and their size	getheap()
total size of objects on PARI stack	getstack()

PARI Types & Input Formats

t_INT . Integers; hex, binary	±31; ±0x1F, ±0b101
t_REAL . Reals	±3.14, 6.022 E23
t_INTMOD . Integers modulo <i>m</i>	Mod(<i>n</i> , <i>m</i>)
t_FRAC . Rational Numbers	<i>n</i> / <i>m</i>
t_FFELT . Elt in finite field F_q	ffgen(<i>q</i>)
t_COMPLEX . Complex Numbers	$x + y * I$
t_PADIC . <i>p</i> -adic Numbers	$x + O(p^k)$
t_QUAD . Quadratic Numbers	$x + y * \text{quadgen}(D)$
t_POLMOD . Polynomials modulo <i>g</i>	Mod(<i>f</i> , <i>g</i>)
t_POL . Polynomials	$a * x^n + \dots + b$
t_SER . Power Series	$f + O(x^k)$
t_RFRAC . Rational Functions	<i>f</i> / <i>g</i>
t_QFI / t_QFR . Imag/Real binary quad. form	Qfb(<i>a</i> , <i>b</i> , <i>c</i> ,{ <i>d</i> })
t_VEC / t_COL . Row/Column Vectors	[<i>x</i> , <i>y</i> , <i>z</i>], [<i>x</i> , <i>y</i> , <i>z</i>]-
t_VEC integer range	[1..10]

t_VECSMALL . Vector of small ints	Vecsmall([<i>x</i> , <i>y</i> , <i>z</i>])
t_MAT . Matrices	[<i>a</i> , <i>b</i> ; <i>c</i> , <i>d</i>]
t_LIST . Lists	List([<i>x</i> , <i>y</i> , <i>z</i>])
t_STR . Strings	"abc"
t_INFINITY . $\pm\infty$	+oo, -oo

Reserved Variable Names

$\pi = 3.14\dots$, $\gamma = 0.57\dots$, $C = 0.91\dots$	Pi, Euler, Catalan
square root of -1	I
Landau's big-oh notation	O

Information about an Object

PARI type of object <i>x</i>	type(<i>x</i>)
length of <i>x</i> / size of <i>x</i> in memory	# <i>x</i> , sizebyte(<i>x</i>)
real precision / bit precision of <i>x</i>	precision(<i>x</i>), bitprecision
<i>p</i> -adic, series prec. of <i>x</i>	padicprec(<i>x</i>), serprec

Operators

basic operations	+, −, *, /, ^, sqr
$i=i+1$, $i=i-1$, $i=i*j$, ...	i++, i--, i*=j,...
euclidean quotient, remainder	$x \backslash y$, $x \backslash y$, $x \% y$, divrem(<i>x</i> , <i>y</i>)
shift <i>x</i> left or right <i>n</i> bits	$x << n$, $x >> n$ or shift(<i>x</i> ,± <i>n</i>)
multiply by 2^n	shiftmul(<i>x</i> , <i>n</i>)
comparison operators	<=, <, >=, >, ==, !=, ==, lex, cmp
boolean operators (or, and, not)	, &&, !
bit operations	bitand, bitneg, bitor, bitxor, bitnegimply
sign of $x = -1, 0, 1$	sign(<i>x</i>)
maximum/minimum of <i>x</i> and <i>y</i>	max, min(<i>x</i> , <i>y</i>)
derivative of <i>f</i>	<i>f</i> '
differential operator	diffop(<i>f</i> , <i>v</i> , <i>d</i> ,{ <i>n</i> = 1})
quote operator (formal variable)	' <i>x</i>
assignment	<i>x</i> = <i>value</i>
simultaneous assignment $x \leftarrow v_1$, $y \leftarrow v_2$	[<i>x</i> , <i>y</i>] = <i>v</i>

Select Components

<i>n</i> -th component of <i>x</i>	component(<i>x</i> , <i>n</i>)
<i>n</i> -th component of vector/list <i>x</i>	<i>x</i> [<i>n</i>]
components $a, a+1, \dots, b$ of vector <i>x</i>	<i>x</i> [<i>a</i> .. <i>b</i>]
(<i>m</i> , <i>n</i>)-th component of matrix <i>x</i>	<i>x</i> [<i>m</i> , <i>n</i>]
row <i>m</i> or column <i>n</i> of matrix <i>x</i>	<i>x</i> [<i>m</i> ,], <i>x</i> [, <i>n</i>]
numerator/denominator of <i>x</i>	numerator(<i>x</i>), denominator

Random Numbers

random integer/prime in $[0, N[$	random(<i>N</i>), randomprime
get/set random seed	getrand, setrand(<i>s</i>)

Conversions

to vector, matrix, vec. of small ints	Col/Vec,Mat,Vecsmall
to list, set, map, string	List, Set, Map, Str
create PARI object (<i>x</i> mod <i>y</i>)	Mod(<i>x</i> , <i>y</i>)
make <i>x</i> a polynomial of <i>v</i>	Pol(<i>x</i> ,{ <i>v</i> })
as Pol , etc., starting with constant term	Polrev, Vecrev, Colrev
make <i>x</i> a power series of <i>v</i>	Ser(<i>x</i> ,{ <i>v</i> })
string from bytes / from format+args	Strchr, Strprintf
TeX string	Strtex(<i>x</i>)
convert <i>x</i> to simplest possible type	simplify(<i>x</i>)
object <i>x</i> with real precision <i>n</i>	precision(<i>x</i> , <i>n</i>)
object <i>x</i> with bit precision <i>n</i>	bitprecision(<i>x</i> , <i>n</i>)
set precision to <i>p</i> digits in dynamic scope	localprec(<i>p</i>)
set precision to <i>p</i> bits in dynamic scope	localbitprec(<i>p</i>)

Conjugates and Lifts

conjugate of a number <i>x</i>	conj(<i>x</i>)
norm of <i>x</i> , product with conjugate	norm(<i>x</i>)
L^p norm of <i>x</i> (L^∞ if no <i>p</i>)	normlp(<i>x</i> ,{ <i>p</i> })
square of L^2 norm of <i>x</i>	norml2(<i>x</i>)
lift of <i>x</i> from Mods and <i>p</i> -adics	lift, centerlift(<i>x</i>)
recursive lift	liftall
lift all t_INT and t_PADIC (\rightarrow t_INT)	liftint
lift all t_POLMOD (\rightarrow t_POL)	liftpol

Lists, Sets & Maps

Sets (= row vector with strictly increasing entries w.r.t. cmp)	
intersection of sets <i>x</i> and <i>y</i>	setintersect(<i>x</i> , <i>y</i>)
set of elements in <i>x</i> not belonging to <i>y</i>	setminus(<i>x</i> , <i>y</i>)
union of sets <i>x</i> and <i>y</i>	setunion(<i>x</i> , <i>y</i>)
does <i>y</i> belong to the set <i>x</i>	setsearch(<i>x</i> , <i>y</i> ,{ <i>flag</i> })
set of all $f(x,y)$, $x \in X$, $y \in Y$	setbinop(<i>f</i> , <i>X</i> , <i>Y</i>)
is <i>x</i> a set ?	setisset(<i>x</i>)

Lists . create empty list: $L = \text{List}()$	
append <i>x</i> to list <i>L</i>	listput(<i>L</i> , <i>x</i> ,{ <i>i</i> })
remove <i>i</i> -th component from list <i>L</i>	listpop(<i>L</i> ,{ <i>i</i> })
insert <i>x</i> in list <i>L</i> at position <i>i</i>	listinsert(<i>L</i> , <i>x</i> , <i>i</i>)
sort the list <i>L</i> in place	listsort(<i>L</i> ,{ <i>flag</i> })

Maps . create empty dictionary: $M = \text{Map}()$	
attach value <i>v</i> to key <i>k</i>	mapput(<i>M</i> , <i>k</i> , <i>v</i>)
recover value attach to key <i>k</i> or error	mapget(<i>M</i> , <i>k</i>)
is key <i>k</i> in the dict ? (set <i>v</i> to $M(k)$)	mapisdefined(<i>M</i> , <i>k</i> ,{& <i>v</i> })
remove <i>k</i> from map domain	mapdelete(<i>M</i> , <i>k</i>)

GP Programming

User functions and closures

x, *y* are formal parameters; *y* defaults to **Pi** if parameter optited; *z*, *t* are local variables (lexical scope), *z* initialized to 1.

fun(<i>x</i> , <i>y</i> =Pi) = my(<i>z</i> =1, <i>t</i>); <i>seq</i>	
fun = (<i>x</i> , <i>y</i> =Pi) -> my(<i>z</i> =1, <i>t</i>); <i>seq</i>	
attach a help message to <i>f</i>	addhelp(<i>f</i>)
undefine symbol <i>s</i> (also kills help)	kill(<i>s</i>)
Control Statements (<i>X</i> : formal parameter in expression <i>seq</i>)	
if $a \neq 0$, evaluate <i>seq</i> ₁ , else <i>seq</i> ₂	if(<i>a</i> ,{ <i>seq</i> ₁ },{ <i>seq</i> ₂ })
eval. <i>seq</i> for $a \leq X \leq b$	for(<i>X</i> = <i>a</i> , <i>b</i> , <i>seq</i>)
...for primes $a \leq X \leq b$	forprime(<i>X</i> = <i>a</i> , <i>b</i> , <i>seq</i>)
...for composites $a \leq X \leq b$	forcomposite(<i>X</i> = <i>a</i> , <i>b</i> , <i>seq</i>)
...for $a \leq X \leq b$ stepping <i>s</i>	forstep(<i>X</i> = <i>a</i> , <i>b</i> , <i>s</i> , <i>seq</i>)
...for <i>X</i> dividing <i>n</i>	fordiv(<i>n</i> , <i>X</i> , <i>seq</i>)
multivariable for , lex ordering	forvec(<i>X</i> = <i>v</i> , <i>seq</i>)
loop over partitions of <i>n</i>	forpart(<i>p</i> = <i>n</i> , <i>seq</i>)
loop over vectors <i>v</i> , $q(v) \leq B$; $q > 0$	forqfvec(<i>v</i> , <i>q</i> , <i>b</i> , <i>seq</i>)
loop over $H < G$ finite abelian group	forsubgroup(<i>H</i> = <i>G</i>)

evaluate <i>seq</i> until $a \neq 0$	until(<i>a</i> , <i>seq</i>)
while $a \neq 0$, evaluate <i>seq</i>	while(<i>a</i> , <i>seq</i>)
exit <i>n</i> innermost enclosing loops	break({ <i>n</i> })
start new iteration of <i>n</i> -th enclosing loop	next({ <i>n</i> })
return <i>x</i> from current subroutine	return({ <i>x</i> })

Exceptions, warnings

raise an exception / warn	error(), warning()
type of error message <i>E</i>	errname(<i>E</i>)
try <i>seq</i> ₁ , evaluate <i>seq</i> ₂ on error	iferr(<i>seq</i> ₁ , <i>E</i> , <i>seq</i> ₂)

Functions with closure arguments / results

select from v according to f `select(f, v)`
apply f to all entries in v `apply(f, v)`
evaluate $f(a_1, \dots, a_n)$ `call(f, a)`
evaluate $f(\dots f(f(a_1, a_2), a_3) \dots, a_n)$ `fold(f, a)`
calling function as closure `self()`

Sums & Products

sum $X = a$ to $X = b$, initialized at x `sum($X = a, b, expr, \{x\}$)`
sum entries of vector v `vecsum(v)`
sum $expr$ over divisors of n `sumdiv($n, X, expr$)`
...assuming $expr$ multiplicative `sumdivmult($n, X, expr$)`
product $a \leq X \leq b$, initialized at x `prod($X = a, b, expr, \{x\}$)`
product over primes $a \leq X \leq b$ `prodeuler($X = a, b, expr$)`

Sorting

sort x by k -th component `vecsort($x, \{k\}, \{fl = 0\}$)`
min. m of x ($m = x[i]$), max. `vecmin($x, \{&i\}$), vecmax`
does y belong to x , sorted wrt. f `vecsearch($x, y, \{f\}$)`

Input/Output

print with/without $\backslash n$, T_EX format `print, print1, printtex`
print fields with separator `printsep(sep, \dots), printsep1`
formatted printing `printf()`
write $args$ to file `write, write1, writetex($file, args$)`
write x in binary format `writebin($file, x$)`
read file into GP `read($\{file\}$)`
...return as vector of lines `readvec($\{file\}$)`
...return as vector of strings `readstr($\{file\}$)`
read a string from keyboard `input()`

Timers

CPU time in ms and reset timer `gettime()`
CPU time in ms since gp startup `getabstime()`
time in ms since UNIX Epoch `getwalltime()`
timeout command after s seconds `alarm($s, expr$)`

Interface with system

allocates a new stack of s bytes `allocatemem($\{s\}$)`
alias old to new `alias(new, old)`
install function from library `install($f, code, \{gpf\}, \{lib\}$)`
execute system command a `system(a)`
as above, feed result to GP `extern(a)`
as above, return GP string `externstr(a)`
get \$VAR from environment `getenv("VAR")`
expand env. variable in string `Strexpand(x)`

Parallel evaluation

These functions evaluate their arguments in parallel (pthreads or MPI); args. must not access global variables and must be free of side effects. Enabled if threading engine is not *single* in gp header.
evaluate f on $x[1], \dots, x[n]$ `parapply(f, x)`
evaluate closures $f[1], \dots, f[n]$ `pareval(f)`
as **select** `parselect($f, A, \{flag\}$)`
as **sum** `parsum($i = a, b, expr, \{x\}$)`
as **vector** `parvector($n, i, \{expr\}$)`
eval f for $i = a, \dots, b$ `parfor($i = a, \{b\}, f, \{r\}, \{f_2\}$)`
...for p prime in $[a, b]$ `parforprime($p = a, \{b\}, f, \{r\}, \{f_2\}$)`
...multivariate `parforvec($X = v, f, \{r\}, \{f_2\}, \{flag\}$)`
declare x as inline (allows to use as global) `inline(x)`
stop inlining `uninline()`

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Linear Algebra

dimensions of matrix x `matsize(x)`
concatenation of x and y `concat($x, \{y\}$)`
extract components of x `vecextract($x, y, \{z\}$)`
transpose of vector or matrix x `mattranspose(x)` or $x \sim$
adjoint of the matrix x `matadjoin(x)`
eigenvectors/values of matrix x `mateigen(x)`
characteristic/minimal polynomial of x `charpoly(x), minpoly`
trace/determinant of matrix x `trace(x), matdet`
Frobenius form of x `matfrobenius(x)`
QR decomposition `matqr(x)`
apply **matqr**'s transform to v `mathouseholder(Q, v)`

Constructors & Special Matrices

$\{g(x) : x \in v \text{ s.t. } f(x)\}$ `[$g(x) \mid x \leftarrow v, f(x)$]`
 $\{x : x \in v \text{ s.t. } f(x)\}$ `[$x \mid x \leftarrow v, f(x)$]`
 $\{g(x) : x \in v\}$ `[$g(x) \mid x \leftarrow v$]`
row vec. of $expr$ eval'ed at $1 \leq i \leq n$ `vector($n, \{i\}, \{expr\}$)`
col. vec. of $expr$ eval'ed at $1 \leq i \leq n$ `vectorv($n, \{i\}, \{expr\}$)`
vector of small ints `vectorsmall($n, \{i\}, \{expr\}$)`
 $[c, c \cdot x, \dots, c \cdot x^n]$ `powers($x, n, \{c = 1\}$)`
matrix $1 \leq i \leq m, 1 \leq j \leq n$ `matrix($m, n, \{i\}, \{j\}, \{expr\}$)`
define matrix by blocks `matconcat(B)`
diagonal matrix with diagonal x `matdiagonal(x)`
is x diagonal? `matisdiagonal(x)`
 $x \cdot \text{matdiagonal}(d)$ `matmuldiagonal(x, d)`
 $n \times n$ identity matrix `matid(n)`
Hessenberg form of square matrix x `mathess(x)`
 $n \times n$ Hilbert matrix $H_{ij} = (i + j - 1)^{-1}$ `mathilbert(n)`
 $n \times n$ Pascal triangle `matpascal($n - 1$)`
companion matrix to polynomial x `matcompanion(x)`
Sylvester matrix of x `polsylvestermatrix(x)`

Gaussian elimination

kernel of matrix x `matker($x, \{flag\}$)`
intersection of column spaces of x and y `matintersect(x, y)`
solve $M * X = B$ (M invertible) `matsolve(M, B)`
as solve, modulo D (col. vector) `matsolvemod(M, D, B)`
one sol of $M * X = B$ `matinverseimage(M, B)`
basis for image of matrix x `matimage(x)`
columns of x *not* in **matimage** `matimagecompl(x)`
supplement columns of x to get basis `mataugment(x)`
rows, cols to extract invertible matrix `matindexrank(x)`
rank of the matrix x `matrank(x)`

Lattices & Quadratic Forms

Quadratic forms

evaluate ${}^t x Q y$ `qfeval($\{Q = id\}, x, y$)`
evaluate ${}^t x Q x$ `qfeval($\{Q = id\}, x$)`
signature of quad form ${}^t y * x * y$ `qfsign(x)`
decomp into squares of ${}^t y * x * y$ `qfgaussred(x)`
eigenvalues/vectors for real symmetric x `qfjacobi(x)`

HNF and SNF

upper triangular Hermite Normal Form `mathnf(x)`
HNF of x where d is a multiple of $\det(x)$ `mathnfmod(x, d)`
multiple of $\det(x)$ `matdetint(x)`
HNF of $(x \mid \text{diagonal}(D))$ `mathnfmod(x, D)`
elementary divisors of x `mathsnf(x)`
elementary divisors of $\mathbf{Z}[a]/(f'(a))$ `poldiscreduced(f)`
integer kernel of x `matkerint(x)`
 \mathbf{Z} -module \leftrightarrow \mathbf{Q} -vector space `matrixqz(x, p)`

Lattices

LLL-algorithm applied to columns of x `qflll($x, \{flag\}$)`
...for Gram matrix of lattice `qflllgram($x, \{flag\}$)`
find up to m sols of **qfnorm**(x, y) $\leq b$ `qfminim(x, b, m)`
 $v, v[i] :=$ number of y s.t. **qfnorm**(x, y) = i `qfrep($x, B, \{flag\}$)`
perfection rank of x `qfperfection(x)`
find isomorphism between q and Q `qfiso(q, Q)`
precompute for isomorphism test with q `qfisoinit(q)`
automorphism group of q `qfauto(q)`
convert **qfauto** for GAP/Magma `qfautoexport($G, \{flag\}$)`
orbits of V under $G \subset \text{GL}(V)$ `qforbits(G, V)`

Polynomials & Rational Functions

all defined polynomial variables `variables()`
get var. of highest priority (higher than v) `varhigher($name, \{v\}$)`
...of lowest priority (lower than v) `varlower($name, \{v\}$)`

Coefficients, variables and basic operators

degree of f `poldegree(f)`
coeff. of degree n of f , leading coeff. `polcoeff(f, n), pollead`
main variable / all variables in f `variable(f), variables(f)`
replace x by y in f `subst(f, x, y)`
evaluate f replacing vars by their value `eval(f)`
replace polynomial expr. $T(x)$ by y in f `substpol(f, T, y)`
replace x_1, \dots, x_n by y_1, \dots, y_n in f `substvec(f, x, y)`
reciprocal polynomial $x^{\deg f} f(1/x)$ `polrecip(f)`
gcd of coefficients of f `content(f)`
derivative of f w.r.t. x `deriv($f, \{x\}$)`
formal integral of f w.r.t. x `intformal($f, \{x\}$)`
formal sum of f w.r.t. x `sumformal($f, \{x\}$)`

Constructors & Special Polynomials

interpolating pol. eval. at a `polinterpolate($X, \{Y\}, \{a\}$)`
 $P_n, T_n/U_n, H_n$ `pollegendre, polchebyshev, polhermite`
 n -th cyclotomic polynomial Φ_n `polcyclo($n, \{v\}$)`
return n if $f = \Phi_n$, else 0 `poliscyclo(f)`
is f a product of cyclotomic polynomials? `poliscycloprod(f)`
Zagier's polynomial of index (n, m) `polzagier(n, m)`

Resultant, elimination

discriminant of polynomial f `poldisc(f)`
resultant $R = \text{Res}_v(f, g)$ `polresultant($f, g, \{v\}$)`
 $[u, v, R], xu + yv = \text{Res}_v(f, g)$ `polresultantext($x, y, \{v\}$)`
solve Thue equation $f(x, y) = a$ `thue($t, a, \{sol\}$)`
initialize t for Thue equation solver `thueinit(f)`

Based on an earlier version by Joseph H. Silverman

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Roots and Factorization

complex roots of f	<code>polroots(f)</code>
number of real roots of f (in $[a, b]$)	<code>polsturm(f, $\{[a, b]\}$)</code>
real roots of f (in $[a, b]$)	<code>polrootsreal(f, $\{[a, b]\}$)</code>
symmetric powers of roots of f up to n	<code>polsym(f, n)</code>
Graeffe transform of f , $g(x^2) = f(x)f(-x)$	<code>polgraeffe(f)</code>
factor f	<code>factor(f)</code>
factor f mod p / roots	<code>factormod(f, p), polrootsmod</code>
... using Cantor-Zassenhaus	<code>factorcantor(f, p)</code>
factor f over \mathbf{F}_{p^a} / roots	<code>factorff(f, p, a), polrootsff</code>
factor f over \mathbf{Q}_p / roots	<code>factorpadic(f, p, r), polrootspadic</code>
cyclotomic factors of $f \in \mathbf{Q}[X]$	<code>polcyclofactors(f)</code>
find irreducible $T \in \mathbf{F}_p[x]$, $\deg T = n$	<code>ffinit(p, n, $\{x\}$)</code>
$\#\{\text{monic irred. } T \in \mathbf{F}_q[x], \deg T = n\}$	<code>ffnbirred(q, n)</code>
p -adic root of f congruent to a mod p	<code>padicappr(f, a)</code>
Newton polygon of f for prime p	<code>newtonpoly(f, p)</code>
Hensel lift $A/\text{lc}(A) = \prod_i B[i] \bmod p^e$	<code>polhensellift(A, B, p, e)</code>
extensions of \mathbf{Q}_p of degree N	<code>padicfields(p, N)</code>

Formal & p-adic Series

truncate power series or p -adic number	<code>truncate(x)</code>
valuation of x at p	<code>valuation(x, p)</code>
Dirichlet and Power Series	
Taylor expansion around 0 of f w.r.t. x	<code>taylor(f, x)</code>
$\sum a_k b_k t^k$ from $\sum a_k t^k$ and $\sum b_k t^k$	<code>serconvol(a, b)</code>
$f = \sum a_k t^k$ from $(a_k/k!)t^k$	<code>serlaplace(f)</code>
reverse power series F so $F(f(x)) = x$	<code>serreverse(f)</code>
Dirichlet series multiplication / division	<code>dirmul, dirdiv(x, y)</code>
Dirichlet Euler product (b terms)	<code>direuler($p = a, b, expr$)</code>

Transcendental and p-adic Functions

real, imaginary part of x	<code>real(x), imag(x)</code>
absolute value, argument of x	<code>abs(x), arg(x)</code>
square/nth root of x	<code>sqrt(x), sqrtn(x, n, $\{\&z\}$)</code>
trig functions	<code>sin, cos, tan, cotan, sinc</code>
inverse trig functions	<code>asin, acos, atan</code>
hyperbolic functions	<code>sinh, cosh, tanh, cotanh</code>
inverse hyperbolic functions	<code>asinh, acosh, atanh</code>
$\log(x)$, e^x , $e^x - 1$	<code>log, exp, expm1</code>
Euler Γ function, $\log \Gamma$, Γ'/Γ	<code>gamma, lngamma, psi</code>
half-integer gamma function $\Gamma(n+1/2)$	<code>gammah(n)</code>
Riemann's zeta $\zeta(s) = \sum n^{-s}$	<code>zeta(s)</code>
multiple zeta value (MZV), $\zeta(s_1, \dots, s_k)$	<code>zetamult(s)</code>
incomplete Γ function ($y = \Gamma(s)$)	<code>incgam(s, x, $\{y\}$)</code>
complementary incomplete Γ	<code>incgamc(s, x)</code>
exponential integral $\int_x^\infty e^{-t}/t dt$	<code>eint1(x)</code>
error function $2/\sqrt{\pi} \int_x^\infty e^{-t^2} dt$	<code>erfc(x)</code>
dilogarithm of x	<code>dilog(x)</code>
m -th polylogarithm of x	<code>polylog(m, x, $\{flag\}$)</code>
U -confluent hypergeometric function	<code>hyperu(a, b, u)</code>
Bessel $J_n(x)$, $J_{n+1/2}(x)$	<code>besselj(n, x), besseljh(n, x)</code>
Bessel I_ν , K_ν , H_ν^1 , H_ν^2 , N_ν	<code>(bessel)i, k, h1, h2, n</code>
Lambert W : x s.t. $xe^x = y$	<code>lambertw(y)</code>
Teichmuller character of p -adic x	<code>teichmuller(x)</code>

Iterations, Sums & Products

Numerical integration for meromorphic functions

Behaviour at endpoint for Double Exponential methods: either a scalar ($a \in \mathbf{C}$, regular) or $\pm\infty$ (decreasing at least as x^{-2}) or $(x-a)^{-\alpha}$ singularity	$[a, \alpha]$
exponential decrease $e^{-\alpha x }$	$[\pm\infty, \alpha]$, $\alpha > 0$
slow decrease $ x ^\alpha$	$\dots \alpha < -1$
oscillating as $\cos(kx)$	$\alpha = k\mathbf{I}$, $k > 0$
oscillating as $\sin(kx)$	$\alpha = -k\mathbf{I}$, $k > 0$
numerical integration	<code>intnum($x = a, b, f$, $\{T\}$)</code>
weights T for intnum	<code>intnuminit(a, b, $\{m\}$)</code>
weights T incl. kernel K	<code>intfuncinit(a, b, K, $\{m\}$)</code>
integrate $(2i\pi)^{-1}f$ on circle $ z-a = R$	<code>intcirc($x = a, R$, f, $\{T\}$)</code>

Other integration methods

n -point Gauss-Legendre	<code>intnumgauss($x = a, b, f$, $\{n\}$)</code>
weights for n -point Gauss-Legendre	<code>intnumgaussinit($\{n\}$)</code>
Romberg integration (low accuracy)	<code>intnumromb($x = a, b, f$, $\{flag\}$)</code>

Numerical summation

sum of series $f(n)$, $n \geq a$ (low accuracy)	<code>suminf($n = a, expr$)</code>
sum of alternating/positive series	<code>sumalt, sumpos</code>
sum of series using Euler-Maclaurin	<code>sumnum($n = a, f$, $\{T\}$)</code>
weights for sumnum, a as in DE	<code>sumnuminit($\{\infty, a\}$)</code>
sum of series by Monien summation	<code>sumnummonien($n = a, f$, $\{T\}$)</code>
weights for sumnummonien	<code>sumnummonieninit($\{\infty, a\}$)</code>

Products

product $a \leq X \leq b$, initialized at x	<code>prod($X = a, b, expr$, $\{x\}$)</code>
product over primes $a \leq X \leq b$	<code>prodeuler($X = a, b, expr$)</code>
infinite product $a \leq X \leq \infty$	<code>prodinf($X = a, expr$)</code>

Other numerical methods

real root of f in $[a, b]$; bracketed root	<code>solve($X = a, b, f$)</code>
... by interval splitting	<code>solvestep($X = a, b, f$, $\{flag = 0\}$)</code>
limit of $f(t)$, $t \rightarrow \infty$	<code>limitnum(f, $\{k\}$, $\{\alpha\}$)</code>
asymptotic expansion of f at ∞	<code>asymnum(f, $\{k\}$, $\{\alpha\}$)</code>
numerical derivation w.r.t x : $f'(a)$	<code>derivnum($x = a, f$)</code>
evaluate continued fraction F at t	<code>contfaceval(F, t, $\{L\}$)</code>
power series to cont. fraction (L terms)	<code>contfracinit(S, $\{L\}$)</code>
Padé approximant (deg. denom. $\leq B$)	<code>bestapprPade(S, $\{B\}$)</code>

Elementary Arithmetic Functions

vector of binary digits of $ x $	<code>binary(x)</code>
bit number n of integer x	<code>bittest(x, n)</code>
Hamming weight of integer x	<code>hammingweight(x)</code>
digits of integer x in base B	<code>digits(x, $\{B = 10\}$)</code>
sum of digits of integer x in base B	<code>sumdigits(x, $\{B = 10\}$)</code>
integer from digits	<code>fromdigits(v, $\{B = 10\}$)</code>
ceiling/floor/fractional part	<code>ceil, floor, frac</code>
round x to nearest integer	<code>round(x, $\{\&e\}$)</code>
truncate x	<code>truncate(x, $\{\&e\}$)</code>
gcd/LCM of x and y	<code>gcd(x, y), lcm(x, y)</code>
gcd of entries of a vector/matrix	<code>content(x)</code>

Primes and Factorization

extra prime table	<code>addprimes()</code>
add primes in v to prime table	<code>addprimes(v)</code>
remove primes from prime table	<code>removeprimes(v)</code>
Chebyshev $\pi(x)$, n -th prime p_n	<code>primepi(x), prime(n)</code>
vector of first n primes	<code>primes(n)</code>
smallest prime $\geq x$	<code>nextprime(x)</code>
largest prime $\leq x$	<code>precprime(x)</code>
factorization of x	<code>factor(x, $\{lim\}$)</code>
... selecting specific algorithms	<code>factorint(x, $\{flag = 0\}$)</code>
$n = df^2$, d squarefree/fundamental	<code>core(n, $\{f\}$), coredisc</code>
recover x from its factorization	<code>factorback(f, $\{e\}$)</code>
$x \in \mathbf{Z}$, $ x \leq X$, $\gcd(N, P(x)) \geq N$	<code>zncoppersmith(P, N, X, $\{B\}$)</code>

Divisors and multiplicative functions

number of prime divisors $\omega(n)$ / $\Omega(n)$	<code>omega(n), bigomega</code>
divisors of n / number of divisors $\tau(n)$	<code>divisors(n), numdiv</code>
sum of (k -th powers of) divisors of n	<code>sigma(n, $\{k\}$)</code>
Möbius μ -function	<code>moebius(x)</code>
Ramanujan's τ -function	<code>ramanujantau(x)</code>

Combinatorics

factorial of x	<code>$x!$ or factorial(x)</code>
binomial coefficient $\binom{x}{y}$	<code>binomial(x, y)</code>
Bernoulli number B_n as real/rational	<code>bernreal(n), bernfrac</code>
Bernoulli polynomial $B_n(x)$	<code>bernpol(n, $\{x\}$)</code>
n -th Fibonacci number	<code>fibonacci(n)</code>
Stirling numbers $s(n, k)$ and $S(n, k)$	<code>stirling(n, k, $\{flag\}$)</code>
number of partitions of n	<code>numbpart(n)</code>
k -th permutation on n letters	<code>numtoperm(n, k)</code>
convert permutation to (n, k) form	<code>permtotnum(v)</code>

Multiplicative groups $(\mathbf{Z}/N\mathbf{Z})^*$, \mathbf{F}_q^*

Euler ϕ -function	<code>eulerphi(x)</code>
multiplicative order of x (divides ϕ)	<code>znorder(x, $\{o\}$), fforder</code>
primitive root mod q / $x \bmod$	<code>znprimroot(q), fprimroot(x)</code>
structure of $(\mathbf{Z}/n\mathbf{Z})^*$	<code>znstar(n)</code>
discrete logarithm of x in base g	<code>znlog(x, g, $\{o\}$), fflag</code>
Kronecker-Legendre symbol $(\frac{x}{y})$	<code>kronecker(x, y)</code>
quadratic Hilbert symbol (at p)	<code>hilbert(x, y, $\{p\}$)</code>

Miscellaneous

integer square / n -th root of x	<code>sqrtint(x), sqrtntint(x, n)</code>
largest integer e s.t. $b^e \leq b$, $e = \lfloor \log_b(b) \rfloor$	<code>logint(x, b, $\{\&z\}$)</code>
CRT: solve $z \equiv x$ and $z \equiv y$	<code>chinese(x, y)</code>
minimal u, v so $ux + yv = \gcd(x, y)$	<code>gcdext(x, y)</code>
continued fraction of x	<code>contfrac(x, $\{b\}$, $\{lmax\}$)</code>
last convergent of continued fraction x	<code>contfracpnqn(x)</code>
rational approximation to x (den. $\leq B$)	<code>bestappr(x, $\{B\}$)k</code>

Characters

Let $cyc = [d_1, \dots, d_k]$ represent an abelian group $G = \oplus (\mathbf{Z}/d_j\mathbf{Z}) \cdot g_j$ or any structure G affording a `.cyc` method; e.g. `idealstar(, q)` for Dirichlet characters. A character χ is coded by $[c_1, \dots, c_k]$ such that $\chi(g_j) = e(n_j/d_j)$.

$\chi \cdot \psi$; χ^{-1} ; $\chi \cdot \psi^{-1}$	<code>charmul, charconj, chardiv</code>
order of χ	<code>charorder(cyc, χ)</code>
kernel of χ	<code>charker(cyc, χ)</code>
$\chi(x)$, G a GP group structure	<code>chareval(G, χ, x, $\{z\}$)</code>

Dirichlet Characters

initialize $G = (\mathbf{Z}/q\mathbf{Z})^*$ $G = \text{idealstar}(q)$
is χ odd? $\text{zncharisodd}(G, \chi)$
real $\chi \rightarrow$ Kronecker symbol (D/\cdot) $\text{znchartokronecker}(G, \chi)$
induce $\chi \in \hat{G}$ to $\mathbf{Z}/N\mathbf{Z}$ $\text{zncharinduce}(G, \chi, N)$

Conrey labelling

Conrey label $m \in (\mathbf{Z}/q\mathbf{Z})^* \rightarrow$ character $\text{znconreychar}(G, m)$
character \rightarrow Conrey label $\text{znconreyexp}(G, \chi)$
log on Conrey generators $\text{znconreylog}(G, m)$
conductor of χ (χ_0 primitive) $\text{znconreyconductor}(G, \chi, \{\chi_0\})$

True-False Tests

is x the disc. of a quadratic field? $\text{isfundamental}(x)$
is x a prime? $\text{isprime}(x)$
is x a strong pseudo-prime? $\text{ispseudoprime}(x)$
is x square-free? $\text{issquarefree}(x)$
is x a square? $\text{issquare}(x, \{\&n\})$
is x a perfect power? $\text{ispower}(x, \{k\}, \{\&n\})$
is x a perfect power of a prime? ($x = p^n$) $\text{isprimepower}(x, \&n\})$
... of a pseudoprime? $\text{ispseudoprimepower}(x, \&n\})$
is x powerful? $\text{ispowerful}(x)$
is x a totient? ($x = \varphi(n)$) $\text{istotient}(x, \{\&n\})$
is x a polygonal number? ($x = P(s, n)$) $\text{ispolygonal}(x, s, \{\&n\})$
is pol irreducible? $\text{polisirreducible}(pol)$

Graphic Functions

crude graph of $expr$ between a and b $\text{plot}(X = a, b, expr)$
High-resolution plot (immediate plot)
plot $expr$ between a and b $\text{ploth}(X = a, b, expr, \{flag\}, \{n\})$
plot points given by lists lx, ly $\text{plothrow}(lx, ly, \{flag\})$
terminal dimensions $\text{plotsizes}()$

Rectwindow functions

init window w , with size x, y $\text{plotinit}(w, x, y)$
erase window w $\text{plotkill}(w)$
copy w to w_2 with offset (dx, dy) $\text{plotcopy}(w, w_2, dx, dy)$
clips contents of w $\text{plotclip}(w)$
scale coordinates in w $\text{plotscale}(w, x_1, x_2, y_1, y_2)$
 ploth in w $\text{plotrecth}(w, X = a, b, expr, \{flag\}, \{n\})$
 plothrow in w $\text{plotrecththrow}(w, data, \{flag\})$
draw window w_1 at $(x_1, y_1), \dots$ $\text{plotdraw}([[w_1, x_1, y_1], \dots])$

Low-level Rectwindow Functions

set current drawing color in w to c $\text{plotcolor}(w, c)$
current position of cursor in w $\text{plotcursor}(w)$
write s at cursor's position $\text{plotstring}(w, s)$
move cursor to (x, y) $\text{plotmove}(w, x, y)$
move cursor to $(x + dx, y + dy)$ $\text{plotrmove}(w, dx, dy)$
draw a box to (x_2, y_2) $\text{plotbox}(w, x_2, y_2)$
draw a box to $(x + dx, y + dy)$ $\text{plotrbox}(w, dx, dy)$
draw polygon $\text{plotlines}(w, lx, ly, \{flag\})$
draw points $\text{plotpoints}(w, lx, ly)$
draw line to $(x + dx, y + dy)$ $\text{plotrline}(w, dx, dy)$
draw point $(x + dx, y + dy)$ $\text{plotrpoint}(w, dx, dy)$
draw point $(x + dx, y + dy)$ $\text{plotrpoint}(w, dx, dy)$

Postscript Functions

as ploth $\text{psploth}(X = a, b, expr, \{flag\}, \{n\})$
as plothrow $\text{psplothrow}(lx, ly, \{flag\})$
as plotdraw $\text{psdraw}([[w_1, x_1, y_1], \dots])$

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