

Modular forms, modular symbols

(PARI-GP version 2.9.0)

Modular Forms

To be completed later.

Modular Symbols

Let $G = \Gamma_0(N)$, $V_k = \mathbf{Q}[X, Y]_{k-2}$. We let $\Delta = \text{Div}^0(\mathbf{P}^1(\mathbf{Q}))$; an element of Δ is a *path* between cusps of $X_0(N)$ via the identification $[b] - [a] \rightarrow$ the path from a to b . A path is coded by the pair $[a, b]$, where a, b are rationals or ∞ , denoting the point at infinity $(1 : 0)$.

Let $\mathbf{M}_k(G) = \text{Hom}_G(\Delta, V_k) \simeq H_c^1(X_0(G), V_k)$; an element of $\mathbf{M}_k(G)$ is a V_k -valued *modular symbol*. There is a natural decomposition $\mathbf{M}_k(G) = \mathbf{M}_k(G)^+ \oplus \mathbf{M}_k(G)^-$ under the action of the $*$ involution, induced by complex conjugation. The `msinit` function computes either \mathbf{M}_k ($\varepsilon = 0$) or its \pm -parts ($\varepsilon = \pm 1$) and fixes a minimal set of $\mathbf{Z}[G]$ -generators (g_i) of Δ .

initialize $M = \mathbf{M}_k(\Gamma_0(N))^\varepsilon$ `msinit(N, k, { $\varepsilon = 0$ })`
the level M `msgetlevel(M)`
the weight k `msgetweight(M)`
the sign ε `msgetsign(M)`

$\mathbf{Z}[G]$ -generators and relations for Δ `mspathgens(M)`
Decompose $p = [a, b]$ on the (g_i) `mspathlog(M, p)`

Create a symbol

Eisenstein symbol attached to cusp c `msfromcusp(M, c)`
Cuspidal symbol attached to E/\mathbf{Q} `msfromell(E)`
symbol having given Hecke eigenvalues `msfromhecke(M, v, { H })`
is s a symbol ? `msissymbol(M, s)`
the list of all $s(g_i)$ `mseval(M, s)`
evaluate symbol s on path $p = [a, b]$ `mseval(M, s, p)`

Operators

An operator is given by a matrix of a fixed \mathbf{Q} -basis. H , if given, is a stable \mathbf{Q} -subspace of $\mathbf{M}_k(G)$: operator is restricted to H .
matrix of Hecke operator T_p or U_p `mshecke(M, p, { H })`
matrix of Atkin-Lehner w_Q `msatkinlehner(M, Q, { H })`
matrix of the $*$ involution `msstar(M, { H })`

Subspaces

A subspace is given by a structure allowing quick projection and restriction of linear operators. Its fist component is a matrix with integer coefficients whose columns for a \mathbf{Q} -basis. If H is a Hecke-stable subspace of $M_k(G)^+$ or $M_k(G)^-$, it can be split into a direct sum of Hecke-simple subspaces. To a simple subspace corresponds a single normalized newform $\sum_n a_n q^n$.
cuspidal subspace $S_k(G)^\varepsilon$ `mscuspidal(M)`
Eisenstein subspace $E_k(G)^\varepsilon$ `mseisenstein(M)`
new part of $S_k(G)^\varepsilon$ `msnew(M)`
split H into simple subspaces (of $\dim \leq d$) `mssplit(M, H, { d })`
(a_1, \dots, a_B) for attached newform `msqexpansion(M, H, { B })`

Overconvergent symbols and p -adic L functions

Let M be a full modular symbol space given by `msinit` and p be a prime. To a classical modular symbol ϕ of level N ($v_p(N) \leq 1$), which is an eigenvector for T_p with non-zero eigenvalue a_p , we can attach a p -adic L -function L_p . The function L_p is defined on continuous characters of $\text{Gal}(\mathbf{Q}(\mu_{p^\infty})/\mathbf{Q})$; in GP we allow characters $\langle \chi \rangle^{s_1} \tau^{s_2}$, where (s_1, s_2) are integers, τ is the Teichmüller character and χ is the cyclotomic character.

The symbol ϕ can be lifted to an *overconvergent* symbol Φ , taking values in spaces of p -adic distributions (represented in GP by a list of moments modulo p^n).

`mspadicinit` precomputes data used to lift symbols. If *flag* is given, it speeds up the computation by assuming that $v_p(a_p) = 0$ if *flag* = 0 (fastest), and that $v_p(a_p) \geq \textit{flag}$ otherwise (faster as *flag* increases).

`mspadicmoments` computes distributions mu attached to Φ allowing to compute L_p to high accuracy.

initialize Mp to lift symbols `mspadicinit(M, p, n, {flag})`
lift symbol ϕ `mstooms(Mp, ϕ)`
eval overconvergent symbol Φ on path p `msomseval(Mp, Φ , p)`
 mu for p -adic L -functions `mspadicmoments(Mp, S, { $D = 1$ })`
 $L_p^{(r)}(\chi^s)$, $s = [s_1, s_2]$ `mspadicL(mu, { $s = 0$ }, { $r = 0$ })`
 $\hat{L}_p(\tau^i)(x)$ `mspadicseries(mu, { $i = 0$ })`

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